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संख्या

EFFEC CYCAS OVULE ROT AT HARDWAR

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(Received 23-12-1989)

गुरुकुल कांगड़ी विश्वविद्यालय पुस्तकालय

ABSTRACT

Present paper deals with the effect of temperature on incidence of Cycas ovule rot at Hardwar. Pathogenesity experiment of Cycas ovule rot due to *Fusarium moniliforme* and *F. moniliforme* var. *intermedium* was conducted at 30°C and 25°C temperature respectively. Larger sized and dark brown coloured lesions were produced by both pathogens at 30°C but lesions were of smaller size and of light brown colour at 25°C. Maximum growth of both pathogens was found at 30°C which was 430 mg (DW), 8.6 cm (diametre of colony) & 570 mg (DW), 7.6 cm. respectively.

Key Words : Cycas, Ovule Symptoms, growth, Pathogen.

INTRODUCTION

Cycas a well known ornamental plant, includes twenty species and grows from Japan to Australia and Madagoskar (Africa). It is a slow growing long lived and evergreen palm like plant. Six species of *Cycas* are found in our country and *C. revoluta* Thenub is the commonest species. Generally female plants of this species are found only in northern India.

The incidence of the disease was observed in the month of October 1986 on the ovules of *C. revoluta* grown in the Botanical garden of

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Gurukul Kangri University Hardwar. Persistence of infection on ovule has been noted upto month of March in year 1987, 88 and 89 regularly. About sixty to seventy percent ovules were infected with the disease. Dry ovule rot of Cycas was a new disease in this country and the pathogen involved in the disease has not been reported from country till now (1,2,4,5). The pathogens were identified as *Fusarium moniliforme* and *Fusarium moniliforme* var. *intermedium* by CMI, Ferryland, Kew surrey, England

As temperature and humidity are very important factors in development of any disease and growth of the pathogen therefore present investigation was undertaken to find out the relationship of temperature with incidence of disease and growth of the pathogen.

MATERIALS AND METHODS

Medium sized ovules showing typical characteristics of the disease (dark brown lesion on ovule surface), were collected from the plants and were brought to the laboratory for isolation of the pathogen and further studies. Ovules were surface sterilized by treating with 0.01% $HgCl_2$ solution for 15 seconds to kill the surface microflora. Treated ovules were repeatedly washed 5 to 6 times with sterilized water. These ovules were cut into small pieces with the help of sterilized blade and were inoculated on PDA plates. Certain ovule pieces were placed on PDA with their inner surface. Inoculated petridishes were incubated in BOD incubater at 26±1°C for growth of the pathogen. The pathogens were purified and stocked on PDA slants. Cultures were sent to CMI and were identified as *Fusarium moniliforme* and *Fusarium moniliforme* var. *intermedium*.

These cultures were used for pathogenecity experiment invitro at 25°C and 30°C temperature. Growth of the pathogens (radial and dry weight) was also studied at above mentioned temperature to see the effect of temperature on development of disease and growth of the pathogens.

Pathogenecity Experiments in Vitro at Different Temperatures

Medium sized healthy ovules showing typical characteristic orange colour were used in the present investigation. The ovules along with megasporophylls were surface sterilized with $HgCl_2$ solution of. 01% for twenty seconds and washed repeatedly with sterilized water. Injuries were made on the ovule surface and injured ovules were inoculated with discs of 8mm. of both pathogens. Discs were taken from edges of six days old colony grown on PDA medium. These inoculated ovules were placed in sterilized enamel trays and were covered with glass beljar. Sterilized wet cotton was placed inside the beljar to provide hundred percent humidity for development of the disease symptoms. Inoculated ovules with both pathogens were incubated at 25°C and 30°C temperature respectively in BOD incubator for development of disease symptoms. Observation was made after 10 days of incubation.

Growth of Pathogens at Different Temperature

The pathogens were grown on potato dextrose liquid broth (PDB) and on PDA medium to find out the effect of temperature on growth of the pathogens. Earlier plated Petridishes were inoculated with the discs of 8 mm. diameter of both pathogens and were incubated in BOD incubatore at 25°C and 30°C for their radial and mycelial growth. The flasks of 250 ml. capacity containing 30 ml. sterilized PDB were inoculated with 8 mm. discs of the pathogens and were incubated at 25°C and 30°C temperature. Diameter of the colony and dry weight of the mycelium was recorded after six days of incubation.

Result and discussion

Temperature has shown profound effect on development of disease symptoms (lesions) on Cycas ovule. It has been found that lesions were much clear, dark brown coloured and larger sized at 30°C than at 25°C in both pathogens i.e. *F. moniliforme* and *F. moniliforme* var. *intermedium* respectively (Table-1).

Maximum diameter of colony as well as dry weight of mycelium in both of pathogens occured at 30°C temperature. Record of dry weight of mycelium showed profound effect of temperature on growth of pathogens (table 2).

Higher growth rate of both pathogens at 30°C is directly related with the incidence of the disease. As both pathogens grew well at 30°C temperature it indicates that pathogens could increase the inoculum potential at 30°C which is an important factor for development of any disease. Day temperature of Hardwar during October and November months was found 30°C in the garden and campus. Symptoms of the disease on Cycas ovule was also observed during October 1986. Luxurient growth of both pathogens at 30°C temperature invitro and incidence of the disease on Cycas ovule at this temperature showed positive correlation in between temperature and disease development. Kulkarni (3) also studied the temperature effects on development of cotton wilt caused by *F. oxysporum, F. vasinfectum* (Atk) Snyder and Hansen and found that 31°C temperature was maximum temperature for disease development.

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TABLE-I

Characterestics of 6ycas ovule rot at different Temperature

Organisms	Incubation Period	Temperature 0°C	Average area of Lesion in cm.	Characterestics of ovule rot
Fusarium moniliforme	10	25	1.00 to 1.5	Generally Circular Semicircular and light in colour
	10	30	2.0 to 3.3	Circular and dark brown colour
F. moniliforme-var. - intermedium	10	25	1.0 to 3.3	Circular and dark brown colour.
	10	30	2.0 to 4.00	Circula and dark brown colours.

TABLE II

Influence of Temperature on growth of *F. moniliforme* and *F. moniliforme* var. *intermedium*

Organisms	Temperature (°C)	Incubation days	Average Diameter of colony in cm.	Dry weight of mycelium in mg.
F. moniliforme	25	6	8.0	400.0
	30	6	8.66	430.00
F. moniliforme var. intermedium	25	6	6.6	552.5
	30	6	7.91	570.0

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EFFECT OF INDUSTRIAL EFFLUENT ON SEED GERMINATION OF SOME AGRICULTURAL CROPS

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ABSTRACT

The present investigation was conducted to find out the effect of industrial effluent of two industries viz. Doon distillery and Indian Drug & Pharmaceutical Limited on seed germination of four agricultural crops i.e. *Triticum vulgare, Luffa cylindrica, Abelmoschus esculentus* and *Phaseolus mungo*. During this study, maximum percentage of seed germination was recorded in wheat seed i.e. 86.6 at 100 per cent concentration of IDPL effluent (E_2). A gradual fall in percentage of seed germination of wheat crop was recorded with dilution of this effluent. Adverse effect of Doon Distillery effluent (E_1) was recorded in seed germination of the all test crops as compared with control.

Key words :– Pollution, Effluent, Germination, percentage.

INTRODUCTION

Water pollution is a major problem of the world. The problem appears quite serious specially for the surface water which receives various kinds of pollutants through various means. It can be stated truly that various larger and smaller stream and other aquatic bodies in the country are in danger. River Ganga is also facing the same problem. It receives various kinds of pollutants in various form i.e. industrial effluent, sewage and other various kinds of garbage throughout the course.

In Rishikesh and Hardwar region, various industries such as IDPL located at Virbhadra, Rishikesh and BHEL may be considered as big source of industrial effluent. Discharge of the effluent of these units into Ganga resulted drastic change in water quality (Shanker [8]. Doon Distillery located at KUANWALA, District Dehradun discharges large quantity of its effluent into Reh. This effluent has indirect effect on water quality of Ganga through river song.

The effect of various kinds of pollutants on water quality in different aquatic system has been studied in the country by various workers (see for instance, [2] and [8]. Albeit the use of sewage for irrigation in agricultural crops is in the practice, but industrial effluent is still discharged either on the earth surface or in aquatic system. Discharge of industrial effluent directly in the streams, not only deteriorates the water quality but has adverse effect on aquatic flora and fauna. Shanker *et al.* [9].

The present investigation was undertaken to find out possible utilization of industrial effluent for irrigation. A large area of agricultural land is found in surrounding areas of IDPL and Doon Distillery, and after positive findings, effluent may be used for irrigation purposes so that the effluents are put to economic use in terms of providing irrigation water to fertilizer.

MATERIALS AND METHODS

Industrial effluents of Doon Distillery and IDPL were collected during 1985 at an interval of one month and were evaluated according to APHA for their certain physico-chemical characters (see) [1]. The effluents were further used for the treatment of seeds of test crops. Four different kinds of seeds i.e. *Triticum vulgare* var. RR-21-, *Luffa cylindrica* var. Faizabadi, *Abelmoschus esculentus* and *Phaseolus mungo* were used as test organisms.

Seeds of the above test crops were surface sterilized with. 1% $HgCl_2$ solution and were repeatedly washed with sterilized distilled water for 6-7 times. After proper washings, these seeds were treated in three different concentrations of each effluent i.e. 100 per cent (Pure effluent), 50 per cent and 25 per cent. Three replicates were used for each concentration. Seeds as such (without treatment) were used for control. Treated seeds were kept in moist chamber, prepared in petridishes of 10 cm diameter with blotting sheets and effluent. Ten seeds were kept in each petridish. These petridishes were kept in room temperature for germination of seed. Percentage of seed germination was recorded after 4 days. Investigation was reconducted three times to confirm the results.

RESULTS AND DISCUSSION

Results of the present investigation have been given in Tabels 1 and 2. Average value of physico-chemical characters of the effluent for the period of a year i.e. 1985 has been given in Table 1. Results revealed that effluent of both industries is highly polluted and contains higher level of total solids, BOD and COD etc. beyond permissible limit (Table 1). Dissolved oxygen was very poor in IDPL effluent and was found always nil in Distillery effluent. Similarly, very low level of pH was recorded in the distillery effluent.

Findings on effect of effluent on seed germination showed that IDPL effluent (E_2) has promotive effect as percentage of seed germination in general except *Phaseolus mungo*. (Table 2). Maximum percentage of seed germination among all the test crops has been recorded in *T. vulgare* i.e. 86.6 at 100 per cent concentration (without dilution). A gradual fall in percentage of seed germination was recorded with dilution of the effluent at 50 and 25 per cent (Table 2). These findings indicate that IDPL effluent contains some nutrients and fulfill the requirement of manures to the germinating and growing crop up to certain extent. Furthermore, the results are encouraging in case of *L. cylindrica* and *A. esculentus* where seed germination was recorded slightly better than the control.

Adverse effect of Doon Distillery effluent (E_1) was recorded on seed germination in all the test crops as compared with control. There was no seed germination at 100% (pure effluent) in *T. vulgare* and *L. cylindrica*, while very poor seed germination was recorded in *P. mungo* and *A. esculentus* (Table 2). Enhancement in percentage of seed germination was found in *A. esculentus* with dilution of effluent at 50 and 25 per cent

(Table 2). However, this did not show any encouragement in case of *T. vulgare* (Table 2).

The poor germination with slight increase with dilution of effluent showed the dilution of various toxic substances which are present in dissolved and suspended particles, BOD and COD etc. as well as this accelerate of pH level of effluent (E_1) i.e. 3.3. Higher concentration of total solids i.e. 25180 mg/lt may be a factor for inhibition of seed germination. Similar finding has been found in seed germination of *Hordium vulgare* by Bahadur and Sharma [3]. Similarly, Sahai and Saxena [7] have also reported harmful effect of effluent on seed germination. Absence of oxygen may also be a cause of inhibition of seed germination in distillery effluent (E_1) as reported by Bahadur and Sharma [3] in *H. vulgare*. Adverse effect of this effluent (E_1) may also be due to another factor i.e. very low level of pH 3.3. As seeds in control were shown in tap water and in general pH of water as found in range 6.8-7.0. The inhibition of seed germination may be due to combined factor rather than any single factor.

Summarily, effluent of IDPL factory (E_2) may be recommended for irrigation for *T. vulgare.*, *L. cylindrica* and *A. esculentus* crops specially for wheat but before to recommend it is highly needed that aforesaid crops should be evaluated for the plant health, yield, fodder quality as well as food and vegetable quality. A biochemical study of the plant and its product is also necessary so that if there is any kind of toxic substances are stored in plant or its product can be estimated quantitatively as well as qualitatively. Albeit in priliminary stage, results of IDPL effluent are promising but cannot be recommended for irrigation till the above information is *not* received experimentally.

TABLE–1

Annual average value of Physico-chemical characteristics of Effluents (1985)

Effluent	pH	Total solids (mg/lt)	D.O. (mg/lt)	BOD (mg/lt)	COD (mg/lt)
E_1	3.3	25180	Nil	7252	15700
E_2	6.9	1200	2.85	189.5	24000

E_1 = Distillery effluent

E_2 = IDPL effluent

TABLE–2

Effect of Effluent on Seed germination of some agricultural crops

Effluent	Treatment (%)	Test crop & Percentage of Germination			
		T. *vulgare*	L. *cylindrica*	A. *esculentus*	P. *mungo*
E_1	100	Nil	Nil	20	0.5
	50	Nil	10	55	17.5
	25	10	43	55	17.5
E_2	100	86.6	56.2	65	Nil
	50	70	59.2	67	Nil
	25	70	36	67	--
Control		53	60	68	75.5

E_1 = Distillery effluent

E_2 = IDPL effluent.

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THEORY OF OLFACTION/GUSTATION BASED ON DIPOLE MOMENT OF MOLECULE

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(Received 21-12-1988 and after revision 10-4-1989)

ABSTRACT

Various Investigators have proposed many theories to explain the phenomena of olfaction [1, 19] and gustation [16], but most of them touched the boundary of the problem. The theory presented in this paper could explain all the aspects of the two phenomena.

The theory helps in understanding the smell and taste (Chemical) senses completley just like there is complete understanding of light and sound senses.

1.00 Introduction : Eyes, ears, nose, tongue and skin are the parts through which sensations are felt by human body. Light and sound sensations felt through eyes and ears have been completely explained by the scientists. However smell and taste sensations felt by the nose and tongue respectively could not be explained satisfactorily by various theories proposed till now. The most popular theory of olfaction has been proposed by Amoore et al. (1) which relates odour quality with shape and size of molecule. Shallenberger et al. [16] proposed a theory of gustation co-relating sweet taste with structure of molecules.

The theory presented in this paper explains completely the two phenomena of chemical senses.

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2.00 Receptors

2.01 Olfactory receptors : There are two types of olfactory receptors; Epithelial and Antennae type. Epithelial type receptors are found in animals, birds, reptiles, fishes etc. and are embedded and flushed with skin surface, usually in their nasal cavity. The general features are similar in them (15). The antennae type receptors are found in insects.

Consider the human olfactory cell as a particular case. The olfactory cell mainly has olfactory bipolar neurone from which cilia emerges. The secretions of Bowmann's glands keep the surface of epithelium moist and furnishes the necessary solvent. The odour molecule reaches the sensitive bipolar neurone after being dissolved in these secretions.. The continuous stream of secretions removes the remains of the stimulating substances and keeps the receptors ready for new odour molecules.

The general features of antennae type receptors of insects are similar. It contains naked dendrites bathed in biological fluid and are surrounded by cuticle having pores. The odour molecules reach sensitive dendritic membrane through these pores, after being dissolved in biological fluid (10).

2.02 Gustatory Receptors : These are groups of taste buds distributed usually on tongue. Each taste bud is a group of cells arranged in a small well around nerve endings. The molecules to be tasted reach these nerve endings by dissolving in water or saliva.

The above facts show that it is the nerve endings which receive odour or taste molecule (hereafter called guest molecule) dissolved in biological fluid. After the molecule has reached nerve endings the phenomena of odour and taste are indentical and further discussions apply equally to both these phenomena.

In case of taste phenomenon, it has been shown that it is not dependent on temperature in the range of 20°C to 30°C (near body temperature), [2] and the same is expected in case of odour phenomenon.

3.00 Molecule and nerve membrane : The guest molecule from air moveș to biological fluid medium in which the nerve endings are bathed and approaches towards the membrane of nerve endings (2.02). The condition of molecule in biological fluid is as described below.

3.01 Structural model of molecule in biological fluid: Since the biological fluids mainly contain water (nearly 80%), [9] a structural model must be formulated on the basis of structure of molecule in water.

The most accepted water hydrate model is being adopted [4]. The water molecules in liquid water exist in monomeric and polymeric forms, forming ice-like structure having cavities, which are capable of accommodating guest molecules. Two types of cavities of sizes 12.03 Å and 17.31 Å diameter have been suggested. The water molecules are hydrogen bonded with each other and have no free electron or proton to make further hydrogen bond. Dimers and trimers are very common.

Most of the gas hydrates constitute a class in which small molecules of many types occupy almost spherical holes in ice-like lattices made up of hydrogen bonded water molecules. The guest molecule will form such types of gas hydrates in water/biological fluid when dissolved in it. The guest molecle remains encaged in the firm enclosure made up of these H-bonded water molecules. The order of water molecules remains same. The guest and water molecules are not held up by any strong force and there is a remarkable degree of rotational mobility on the part of encaged molecule behaving as if in a vapour state [7]. The interaction of water and guest molecule appears primarily of van der Wall's type even for polar guest molecule. Guest molecule upto 5 Å in size fit easily into cages while a larger molecule, which can not be accommodated in a single cavity may occupy several adjacent sites.

The above model has been adopted for the interaction of guest molecule with nerve membrane. Before discussion of the interaction, some points regarding membrane and its excitation are discussed in the following para.

4.00 Membrane and excitation:

4.01 Membrane protein : It has been established that in the membrane, the lipid bilayer forms a barrier while its protein molecules after suffering conformational change are responsible for excitation. The exact mechanism, of excitation has not been understood. Thus, it is concluded that the guest molecule present at the outer side of nerve membrane, interacts with its protein, causing conformational change in it, resulting the excitation [11].

The globular protein of membrane contains 30% to 100% α-helix besides other parts. There is no water inside α-helix of protein. It has a delicately balanced structure. It performs highly specific and complicated chemical actions, which require high degree of organisation. In living organism it exists in native state. We have very little knowledge about its native or denatured state. The incorporation of certain proteins and polypeptides in artificial lipid bilayer may acquire selectivity, electrical excitability and even chemical receptor properties.

Protein molecules have two types of bonds; strong (Co-valent) and weak (Non-covalent) bonds. The weak hydrogen bonds are the predominent bonds governing folding and unfolding of protein.

Since no monopoles are observed inside the protein, the interactions of the multipoles are short and may be restricted to close neighbours only, in case of energy calculations.

Therefore, if the native protein is under stress, which increases continuously from zero stress (native state) to a higher value, the weak hydrogen bonds will break first resulting in its denatured state. This is a important conclusion for the present studies. Breaking of H-bond in protein of retina has been taken as primary cause of vision perception (14).

4.02 Model of membrane protein : A model of protein molecule has to be built up for further studies which should fulfil the excitation phenomenon. The main features of the proposed model are :

(a) Membrane excitation is due to conformational changes in the protein molecule.

(b) Structure, formation and stability of protein molecule depend mainly on non-covalent bondings.

(c) Hydrogen bonds are most important non-covalent bonds responsible for conversion of protein from native to denatured state which results in excitation in membrane.

Major portion of protein molecule comprises of right handed α-helix. It has been speculated that a reversible transition (from α-helix to sheet structure) may be the cause of muscle contraction and other types of movement in living organism. In a number of enzymes the biological activity depends on α-helical content (5). Thus it is concluded that :

(a) The globular protein is a delicately balanced structure and a small change in its energy may lead to a large change in its overall structure.

(b) Breaking of H-bond of α-helix, leading to conformational changes in protein molecule is the most probable cause of excitation.

(c) α-helix are present almost always on the surface of the protein [3].

On the basis of above facts, the following model has been adopted for further studies. In this model the polypeptide chain of protein is folded into α-helix. Th folding is maintained by H-bonds between -NH and =CO groups. Helix are assumed to be held rigidly with each other by di-sulphide bonds at some places (Fig. 1).

5.00 Interaction of guest molecule with protein : The Dipole Moment (D.M.) :

Interaction of guest molecule with protein molecule results in biological activity. Important facts regarding the interaction are as below:

(i) Guest molecules, under consideration may be taken as small molecule which neither react with water nor ionise in it e.g. O_2 CH_3, OH, C_4 H_9 NH_2 etc.

(ii) Protein molecule has α-helical content as a major portion bathed in water medium. It is inpenetrable to solvent and is surrounded by a firm layer of water of uni-molecular thickness [9].

(iii) The guest molecule when dissolved in water, lies in the interstitial cavity of water medium and forms a hydrate with it. The concentration of guest molecue is infinitly low ($< 10^{-3}$ M), [18] so that it can be considered as an isolated molecule, not affected by the presence of other similar molecules.

(iv) In a state of infinite dilution, the structure of guest molecule does not change and can be compared with its structure in the gaseous state [4]. There is very weak coupling between guest molecule and surrounding water molecules. At low concentrations any marked effects normally represent specific interactons of co-solute (guest molecule) with protein [13].

(v) The protein molecule consists of various bonds out of which H-bonds are weakest. Any small strain on these H-bonds may result in a major change in the protein molecule because it has a delicately balanced structure. Since denaturation process has been split into two step transition. Firstly native to a weekly co-operative transition and then to a state of denaturation (6).

We are interested in the strain produced on H-bond due to the presence of guest molecule which may result into complete danaturation process or a part of it as suggested above.

Other reasons for consideration of H-bonds of α-helix are:

1. α-helix are most abundent, delicately balanced structure with H-bonds continuously breaking and forming. They are found at the surface of protein.

2. Transition of α-helix to β-configuration may be responsible for biological activity which is the main consideration.

(vi) Now suppose a guest molecule 'M' when dissolved in water (or biological fluid), is approaching the helical part of protein molecule. When it is at the closest state to protein molecule, the situation is as shown in fig. 1. This type of bonding is reversible (8).

(vii) In the situation as depicted in para (vi), the stability of a H-bond due to its interaction with guest molecule 'M' (Say CH_3 OH) is to be investigated. A further Simplification gives the pictute (fig. 2) with only one H-bond and part of guest molecule in adjacent cavity. Here interactions except guest molecule and H-bond are neglected (17).

(viii) When there is no guest molecule 'M', the H-bond is stable with cage structure of water medium not occupied. The only difference between this situation and the situation depicted in Fig. 2 is that the cage structure in former is not occupied by a guest molecule, while in later case it is occupied.

(ix) There are two types of cages having exterior sizes equal to 12.03 Å and 17.31 Å giving an average exterior size as 14.68 Å. The average internal size is 4.2 Å.Thus the average thickness of cage is 3.14 Å.Take the distance of H-bond from unimolecular water layer as 1 Å. In case of polar guest molecule its polar part is most likely to be nearer to H-bond (Fig. 2 for CH_3 OH Molecule) and its minimum distance from H-bond will be little more than $1.52 + 3.14+1.0 = 5.66$ Å Say 6 Å. Thus the dipolar part of guest molecule can not come closer to H-bond than this value. If the molecule is bigger and can not be accommodated in one cavity, its polar part will lie nearer to H-bond and non-polar part will occupy adjacent cavities.

(x) When 'M' has entered the hydrate cage, it will interact electrostatically with H-bond.

(xi) The interaction energy between 'M' and H-bond is to be estimated. This is an additional energy due to presence of 'M' and will strain H-bond. If the interaction is of attractive in nature, the energy is surplus.

(xii) The interactions are due to di-or multipoles and therefore are short range interactions. This may be restricted to close neighbours only.

(xiii) The approximate maximum surplus strain energy experienced by H-bond due to presence of 'M' (Fig. 2) can be computed by Stockmeyer's potential. This energy,

$$E = 4\,\varepsilon\left[\left(\frac{\sigma}{\gamma}\right)^{6} - \left(\frac{\sigma}{\gamma}\right)^{12}\right] - \frac{2\,\mu_1\,\mu_2}{\gamma^3} \quad (\rightarrow\rightarrow)$$

(xiv) If the distance of separation 'r' between 'M' and H-bond is medium (6 Å to 15 Å) the 1st part of the equation (i) is quite low as compared to IInd part and energy E can be approximated to:

$E = \dfrac{2\mu_1\,\mu_2}{D^{*} r^{3}}$, where 'D*' is the dielectric constant.

(xv) D.M. of H-bond 'u' is a fixed quantity, thus

$$E \,\alpha\, \frac{\mu_2}{D^{*} r^{3}}$$

(xvi) If the protein molecule and medium remain same (as in case of nerve membrane) and value of 'r' is constant (in case of closest approach of polar part of 'M' to H-bond), the energy is dependent on 'μ_2' i..e. D.M. of guest molecule only.

(xvii) This (excess) energy will strain H-bond and may break it, if it is more than the bond energy of H-bond.

(xviii) Any strain to H-bond may result in drastic changes to the structure of α-helix which has a delicatly balanced structure. The Co-valent bonds of α-helix may act as an amplifier in magnifying the small changes in its structure into re-organisation of the molecule on a larger scale.

(xix) The dielectric constant 'D*', has been assumed to vary with 'r' in a linear manner with values as below :

r	o Å	6 Å	30 Å
D*	1	2	80

5.01 Hydrogen bond and interaction energy : Various investigators have given the following values of H-bond energy ; 0-2 K cal/mole, 100 cal/mole. A lower value of the bond energy has been suggested in aquous medium [6]. A value of 100 cal/mole seems to be a reasonable estimate.

As a typical case, interaction energy has been calculated for the case of interaction of CH_3 OH molecule with H-bond. The values of parameters are :-

CH_3 OH (molecule)		H-bond
value of 'ε'	0.82 K cal/mole	0.073 K cal/mole
value of 'σ "	3.69 Å	3.0 Å
value of 'μ'	1.7 D	1.3 D

Taking value of r=6 Å and dielectric constant D*=2, the total energy is nearly 90 cal/mole. The dipole-dipole interaction energy is 84% of total energy and this % increases with increase in value of 'r'.

Thus it is seen that the interaction energy is of the order of H-bond energy. It should also be kept in mind that the nature performs its functions in a more efficient manner and may need lesser energy than what has been estimated (see also para 5.00 v).

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5.02 Conclusions : It has been seen that the guest molecule is capable of straining (or breaking) H-bonds, which brings a major change in protein of dendritic membrane, resulting in biological activity. The strain caused to H-bond is proportional to the Dipole Moment of guest molecule only (see para 5.00). Thus it is concluded that the Dipole Moment of guest molecule may be related directly to the biological activity of dendritic membrane of olfactory/gustatory receptors causing typical odour/taste.

With these ideas a plot of molecular Dipole Moment and its characteristic odour and taste (called odour/taste spectrum) has been proposed to check the validity of above conclusion.

6.00 THE SPECTRUM

6.01 Description of Odour and Taste: Odour of many molecules have been described by investigators in a variety of ways and it is little difficult to pin-point the correct statement. Therefore, a personal judgement has been applied in such cases in assigning a typical odour for these molecules. However, many molecules have been left out of this study where the odour description has not been precise e.g. 'sharp penetrating', 'mild odour', 'pungent' etc. Only those molecules have been selected where their odours have been described as typical odour e.g. 'rose odour', 'pear odour' etc.

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Description of taste of molecules is even lesser in number and less descriptive. The procedure followed in case of odour molecule has been adopted in selecting the taste of a molecule.

6.02 Intensity : It has been felt that intensity of odour/taste depends upon the rate of molecules reaching the membrane of receptors, which depends upon the physical parameters such as wind velocity, temperature, rate of out flow of molecules from the source, its water solubility etc. It has been observed that odour/taste of a molecule changes due to wide variations in its intensity. This may be due to some different changes in membrane protein because of presence of molecules in higher number than what is considered here i.e. in infinitly low number. In present studies this factor of higher intensity has not been considered.

6.03 Odour/Taste Molecule: Most of the molecules under consideration exist in monomeric forms e.g. halo-forms, highly halogenated compounds, acytylenes, ketones, ethers, esters, olefines, aromatic and tertiary amines, nitriles, isonitriles, saturated hydrocarbon, carbon di-sulfide etc.

There are molecules which self associate to give a bigger molecule e.g. phenols, alcohols, inorganic and carboxylic acids, primary and secondary amines etc. Even these molecules exist in monomeric forms if concentration is less than 10^{-3} M.

The molecules falling in above categories have been considered in this study. Molecules reacting in atmosphere or reactng with membrane components have been excluded from these studies.

6.04 Value of Dipole Moment : Due to following reasons difficulty has been felt in selecting the value of D.M. of a particular molecule.

(a) These values have been reported for gas form as well as in solutions of various solvents at different temperatures. The solvent effect of non-polar solute has been found to be negligible (12). However, gas values near body temperature has been selected as most appropriate value.

(b) In many cases very few values are available and it is difficult to assess the correctness of the reported value (s). Such values have been used with some restraint.

6.05 Odour/Taste Spectrum: On the basis of conclusion drawn in para 5.02 that the **molecular dipole moment is directly related to its characteristic odour/taste;** a spectrum of Dipole Moment expressed in Debye unit 'D' and the characteristic odour/taste of the molecules has been proposed (See Annexure). Its general features are:

(i) The plot ranges from D.M. = 0 to 3.86 D

(ii) The spectrum is applicable in case of odour/taste molecules present in low concentrations.

(iii) There are 113 distinct odours and 48 distinct tastes.

(iv) Molecules with D.M. < 0.15 D are odour/tasteless. This indicates that the interaction energy in such cases is not enough to cause excitation to the extent that it is perceptible to human organs.

(v) The spectrum is broadly divided into following parts.

D.M. = 0 to 0.15 D-odour/tasteless
= 0.15 D to 0.4 D-Hydrocarbon odour/taste
= 0.4 D to 0.65 D-Oily odour/taste
= 0.65 D to 1.75 D-Mixed odour/taste
= 1.75 D to 1.95 D-Fruity, Flowery odour/taste
(values close to water)
= 1.95 D to 2.40 D-Animal odour/taste
= 2.40 D to 3.56 D-Perfumery odour/taste.

(vi) Water occupies a special status. It is odour/tasteless even though its D.M. value is very high. It is because when water molecule reaches near membrane which is already bathed in water medium, the situation near it does not change at all. Therefore, water is odour/tasteless.

7.00 Conclusions : The various conclusions drawn from present studies are listed below:

(i) The excitation in olfactory/gustatory organ is due to interaction of odour/taste molecule with its sensitive membrane.

(ii) Molecules having same D.M. have similar odour/taste, while molecules of different D.M. have different odour/taste. This verifies the inference drawn in para 5.02 that D.M. is directly related to the biological activity of odour/taste molecules.

(iii) Same value of D.M. is representative of both odour and taste of the molecule simultaneously e.g. D.M. = 1.44 D corresponds to acetic acid odour as well as acetic acid taste.

(iv) The molecules of a group of hydrocarbon having same polar part but different chain length have similar odour/taste. It is because the polar part is nearer to H-bond of membrane protein while the tail is away and the effect of tail is very less as compared to its polar part.

(v) The resolution power of human (Olfactory/Gustatory) receptor membrane in terms of D.M. is 0.01 D i.e. the membrane in both cases is capable of distinguishing molecules if their D.M. differ by more than 0.01 D.

(vi) The general conclusions drawn from studies of human receptors seems to apply in case of receptors of other vertebrates, insects, birds etc.. because there seems to be no reasons contradictory to it in light of para 2.00. Here it is emphasized that human olfactory sense is quite inferior to many animals and insects.

(vii) Pleasent or unpleasentness depends upon individual nature. Normally all odour/taste, felt at low concentrations are pleasent and are unpleasent at high concentrations.

(viii) The spectrum will help in predicting odour and taste of a molecule of known value of D.M. and vice-versa. Knowledge of D.M. will help in finalising molecular structure of a molecule.

(ix) The spectrum will be helpful in finding out new synthetic flavours.

ANNEXURF

Odour/Taste Spectrum O = Odour

T = Taste

D.M. (D)	Odour/Taste	D.M. (D)	Odour/taste.
0.00	Odourless/Tasteless	0.85	Iodoform (O)
0.10	Odourless/Tasteless	0.87	Chloral (O)
0.15	Nitrous Oxide (O)	0.90	Rotten egg (O)
0.20		0.95	Chloroform (O, T)
0.25		1.00	Grapes (O, T)
0.30		1.01	Orange blossom (O)
0.35	Hydrocarbon (O)	1.02	
0.40	Kerosine (O)	1.03	Metallic (O)
0.45	Tarpene (O)	1.04	
0.475	Benzene (O)	1.05	Gerfanium (O)
0.50	Fried Potato (O..T.)	1.06	1.68
0.55		1.065	Napthalene (O)
0.56	Ozone (O)	1.08	Fishy (O)
0.60		1.10	Halogen (O)
0.65		1.11	
0.70		1.12	
0.75		1.13	Chloroethynyl Benzene (O)
0.80		1.14	

1.15		1.46	
1.16	Diethyl ether (O)	1.48	
1.17	Sweet (T)	1.49	Ammonia (O)
1.18	1.805	1.50	
1.19	Caproic (O, T)	1.51	Cabbage/radish (O,T)
1.20		1.52	
1.21		1.53	Sulfuraceous/Shunk (O)
1.22		1.54	
1.23	Piperidine (O)	1.55	Thyme (O)
1.24		1.56	
1.25	Anisole (O)	1.565	Peppery (O)
1.26		1.575	Woody (O, T)
1.28		1.58	
1.30	Marigold/wall flower (O)	1.585	Dry tea leaves (O)
1.32		1.595	Ylang-Ylang (O)
1.34	Hyacinth (O)	1.60	Narcissus (O)
1.36	Earthy (O)	1.605	Lily (O)
1.37	Hazelnut (O, T)	1.61	Jasmine (O)
1.38		1.62	Orrissy (O)
1.40	Rancid Buttery Cheesy (O, T)	1.64	Dry leather (O)
1.42		1.66	Phenolic/medicinal (O, T)
1.44	Acetic acid (O,T)	1.67	Chemical (O, T)

1.685	Fusal oil (O)	1.89	Anisic (O)
1.70	Alcohlic (O, T)	1.88	Strawberry (O, T)
1.72	Mandrain(Citrusy) (O, T)	1.90	
1.73	Lemon (O, T)	1.91	Oily burnt (O.T.)
1.74	Orange (O)	1.92	Rosemerry/Lavandin
1.76	Waxy (O)	1.93	Piney (O)
1.77	Honey (O, T.)	1.94	
1.78	Rose (O, T)	1.96	
1.79		1.98	
1.80	Apple (O,T) Pear (O, T)	1.99	Onion (O.T.)
1.81	Banana (O, T)	2.00	Garlic (O.T.)
1.815	Pineapple (O, T)	2.02	
1.82	Apricot (O, T)	2.04	
1.83	Blackcurrent (O, T)	2.06	
1.835	Water (odour & Tasteless) (O, T)	2.07	Orange leaf/flower (O)
1.84		2.08	
1.845	Ashtray (O)	2.12	
1.85		2.14	Animal (O, T)
1.86	Brandy (O)	2.155	Indolic (O)
1.87	Rum (O, T)	2.16	

2.17	Tarry (O)
2.18	
2.19	Nitric acid (O)
2.20	
2.21	Burnt/smoky (O, T)
2.22	
2.24	Buttery (O, T)
2.25	Coconut (O, T)
2.26	
2.28	Earthy (O)
2.30	Oily (O)
2.32	Walnuts (O)
2.34	Soapy (O)
2.36	
2.37	1, 5 di-chloropentane (O)
2.38	
2.40	
2.42	
2.44	
2.45	Vanilla (O.T.)
2.46	Chocolate (O)
2.475	Winey (O)
2.48	
2.50	
2.51	Wintergreen (O.T.)
2.52	
2.54	
2.55	Clove Oil (O.T.)
2.56	
2.58	Peach (O.T.)
2.60	
2.62	
2.63	Musty (O)
2.64	
2.65	Gassy (O)
2.66	
2.67	Tobacco leaves (O)
2.68	
2.70	
2.72	Roasted coffee (O)
2.74	
2.75	Bitter almond (O.T.)
2.76	
2.78	Musky (O.T.)
2.80	Mushroom (O)
2.82	

2.84			
2.85	Camphor (O)	3.16	Hawthorn (O)
2.86		3.18	
2.88	Peppermint (O.T.)	3.20	
2.90		3.22	
2.91	Acetone (O.T.)	3.24	
2.92		3.26	
2.94		3.28	
2.95	Balsamic (O)	3.30	Hay (O)
2.96		3.32	
2.98	Burning fat (O)	3.34	
3.00		3.36	
3.01		3.38	
3.02		3.40	
3.03		3.42	
3.04		3.44	
3.05	Mimosa (O)	3.45	Cederwood (O.T.)
3.06		3.46	
3.07		3.48	
3.08		3.50	
3.09		3.52	
3.10	New mown hay (O)	3.54	Heliotropine (O.T.)
3 12		3.56	

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3.14		3.58	Benjoin (O)
3.60		3.80	Caramellic (O.T)
3.62		3.82	
3.64		3.84	
3.66	Bread (O.T.)	3.86	Orris/Ambre/vetiver (O)
3.68		3.88	
3.70	Cinnamon (O.T.)	3.90	
3.72		3.92	
3.74		3.94	
3.75	Spicy (O.T.)	3.96	
3.76		3.98	
3.78		4.00	

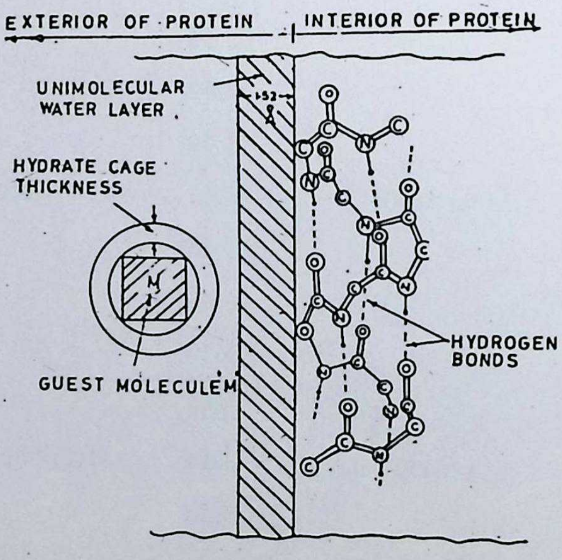
EXTERIOR OF PROTEIN
INTERIOR OF PROTEIN
UNIMOLECULAR WATER LAYER
1.52 Å
HYDRATE CAGE THICKNESS
M
GUEST MOLECULE M
HYDROGEN BONDS

FIG.-I HELIX AND GUEST MOLECULE

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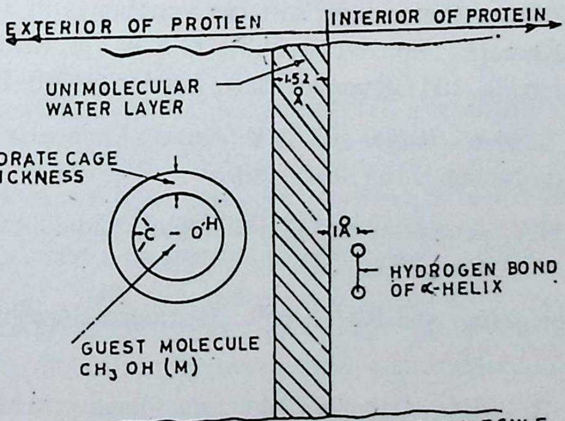
EXTERIOR OF PROTIEN
INTERIOR OF PROTEIN
UNIMOLECULAR WATER LAYER
1.52 Å
HYDRATE CAGE THICKNESS
C – O H
1Å
HYDROGEN BOND OF α-HELIX
GUEST MOLECULE CH₃ OH (M)

FIG-2 HYDROGEN BOND AND GUEST MOLECULE

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A STUDY OF ESSENTIAL OIL FROM ERIGERON LINIFOLIUS WILLD

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ABSTRACT

Essential oil from *Erigeron linifolius* willd, a widely distributed plant in Kumaon and Garhwal regions, possessed

$$d^4_{20°}=0.8782\ ;\ n^D_{20°}=1.53\ ;\ \alpha^D_{20°}=41°\,21'$$

Acid value = 0.3 ; Ester value = 50.2 and Carbonyl value = 0.8. GLC study of the oil using Honeywell HP 7620–A model showed the presence of α-pinene, β-pinene, myrcene, ocimene, limonene, caryophyllene, humulene, cubebne and lachrophyllum ester.

Key words and phrases : Essential oil, *Erigeron linifolius Willd. Chemistry-subject classification* : Pharmaceutical Analysis Essential oils-62.

Erigeron linifolius Willd, belonging to N.O. Compositae, is widely distributed in Kumaon and Garhwal regions. Leaves of the plant are used in Malaya for rheumatism, lumbago and poulticing [1]. Chemical composition of essential oils from *Erigeron annus, Erigeron karwinsky anus* DC, and *Erigeron philadiphicus* [2] [3] [4] have been reported but very little work is reported on essential oil of *Erigeron linifolius* Willd.

Experimental : The essential oil was obtained by water-steam distillation of the plant in a copper still, extracting the distillate with petroleum ether (b 60-80°), drying over sodium sulphate and concentrating the organic phase under reduced pressure (yield = 0.2%). Physico-chemical properties of the oil were :

Sp.gravity/$_{20}$°=0.8782 ; Ref. index/$_{20}$° = 1.53 ; Sp. rotation/$_{20}$° = +41°21'

Acid value = 0.3 ; Ester value = 50.2 and Carbonyl value as $C_{10}H_{16}O$ (oximation) = 0.8.

Fine needle shaped crystals formed, on keeping the oil overnight during winters, were separated. The process was repeated till formation of crystals ceased. Hydrocarbon and Oxygenated fractions of the solid free oil were separated by using a column packed with silicic acid and n-hexane as eluting agents [5]. The two fractions were subjected to GLC examination, separately, under similar parameters. Different constituents of the fractions were identified by injection method and comparison of retention times under the same operating parameters using the pure compounds. The conditions of the GLC analysis were :

Column	..	Carbowax 20 M, 10%.
Carrier gas	..	Nitrogen (30ml per minute)
Column temperature	..	200°
Injection temperature	..	260°
Detector type	..	Thermal conductivity detector (temperature 300°)
Chart speed	..	0.25" per minute.
Sensitivity	..	10
Chromatograph model	..	Honeywell HP 7620-A.

Results of GLC of Hydrocarbon fraction (F1) and Oxygenated fraction (F2) are summarised in table 1 and table 2, respectively.

The solid fraction, separated from the oil earlier was further purified by column chromatography and TLC. Solvent systems used were chloroform, petroleum ether-pyridene (95:5). After development the plates were sprayed with fluorescein-bromine reagent for visual identification. Better resolution was obtained with petroleum ether pyridene. Two fractions, isolated, in pure form, were identified as cis-lachrophyllum ester and matricaria ester by Co-TLC and spectroscopic examination.

Spectroscopic examination of the fractions :

Fraction 1:

The fraction melted at 32°C and possessed molecular formula $C_{11}H_{10}O_2$ (C=75.8%, H=5.74% and 0=18.46%, Molecular weight = 174).

UV : λ max at 218 and 308 nm indicated the presence of symmetrical diene and three conjugated system units, respectively

IR : bands at 3650 cm^{-1} –3100 cm^{-1} (–CH–stretching); 2800 cm^{-1} 3000 cm^{-1} (CH_3–CH_2_CH_2C≡C–); 1600 cm^{-1} (strong) for carboxylate ion; 1750 cm^{-1} for α–β–unsaturated ester; 2200 cm^{-1} (medium) for –C ≡ C–; 970 cm^{-1} for –CH≡CH– and 1600 cm^{-1} for symmetrical diene.

IR spectrum of the compound was found the finger print of IR spectrum of matricaria ester.

Fraction 2 : The fraction melted at 33°C and possessed molecular forumula $C_{11}H_{12}O_2$ (C = 75.0%, H = 6.8% and 0 = 18.2%. Molecular weight = 176).

UV : λ max at 219 nm (for –C≡C–C≡C– unit) and 275 nm (for two conjugated system unit).

IR : bands at 3650 cm^{-1} –3100 cm^{-1} (–CH stretching); 2800cm^{1}– 3000 cm^{-1} (CH_3– C H_2– C H_2– C ≡ C–) ; 1600 cm^{-1} (strong) for carboxylate ion; 1445 cm^{-1} (medium) for – CH_2–C≡C– ; 1400 cm^{-1} (weak) for carboxylate ion; 1750 cm^{-1} for α–β–unsaturated ester; 2200 cm^{-1} (medium) for –C≡C– ;970 cm^{-1} for –CH=CH– and 1600 cm–1 for symmetrical diene.

IR spectrum of isolated lachrophyllum ester was found the finger print of IR of the pure sample of the ester.

PMR : A triplet at 1.0 (δ) for $-CH_3$; a hextet at 1.57 (δ) for $-CH_2-$ $(CH_2-CH_2-CH_2-)$; at 6.13 (δ) for protons of $>C-C<^{COOR}_{H}$ and a singlet at 3.75 (δ) for $-OCH_3$.

Combustion analysis, UV, IR and PMR results correspond to the following structure :

$(CH_3-CH_3-CH_2-(C\equiv C-)_2\ CH=CH-COOCH_3$

The oil was also studied for antibacterial and antifungal properties by 'Filter paper disc diffusion technique' following Difco Laboratories Manual (6). The oil showed inactivity against *E. coli* and *Tricophyton mentdgrophytes* . It showed slight activity against *S. Aureus*.

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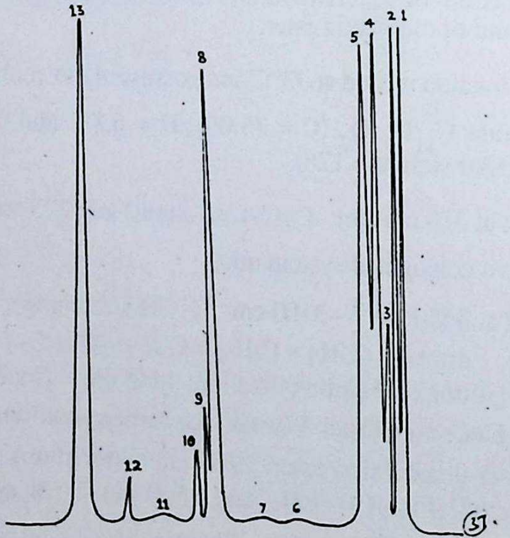


F 1 : GLC OF HYDRO CARBON FRACTION

TABLE - 1

GLC of Hydrocarbon fraction

Peak No.	Compound identified	Mode of identification
1.	α-pinene	IR, T_R
2.	β-pinene	IR, T_R
3.	myrcene	IR, T_R
4.	ocimene	IR, T_R
5.	limonene	IR, T_R
6.	u.i.	
7.	u.i.	
8.	caryophyllene	IR, T_R
9.	u.i.	
10.	humulene	IR, T_R
11.	u.i.	
12.	u.i.	
13.	cubebne	IR, T_R

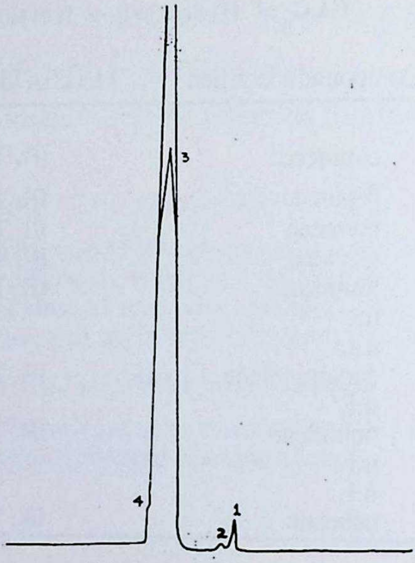
In this table u.i. denotes unidentified peaks.
IR denotes Infra red spectroscopy.
T_R denotes relative retention times.

TABLE - 2

GLC of Oxygenated fraction

Peak No.	Compound identified	Mode of identification
1.	u. i.	
2.	u. i.	
3.	lachrophyllum ester	IR, T_R
4.	u. i.	

In this table u.i. denotes unidentified peaks.
IR denotes Infra red spectroscopy.
T_R denotes relative retention times.



F 2: GLC OF OXYGENATED FRACTION

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ON CONVEXITY IN CONVEX METRIC SPACES WITH APPLICATION

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1. ABSTRACT

In this paper we prove a generalization of a fixed point theorem from [7] for Takahashi convex metric spaces. For this class a characterization of convex hull is given too.

Key words and phrases: Convex metric space, fixed point.
AMS (MOS) Subject Classifications (1980) : 47H10.

2. INTRODUCTION

In 1970 Takahashi [5] introduced the definition of convexity in metric space and generalized some important fixed point theorems previously proved for Banach spaces. Subsequently, Machado [3], Talman [6], Naimpally and Singh [4], Guay and Singh [1], Hadžiac′ and Gajic′ [2], among others have obtained additional results in this setting.

This paper is a continuation of these investigations. Without additional conditions on convex structure we are able to give a characterization of convex hull. The major fact here is that the diameter is invariant under passage to so defined convex hull. In second part, for Takahashi convex metric spaces, we prove a generalization of a fixed point theorem from [7].

3. PRELIMINARIES

DEFINITION 1. Let X be a metric space and I be the closed unit interval. A mapping $W: X \times X \times I \rightarrow X$ is said to be a convex structure on X if for all $x, y \in X, \lambda \in I$,

$$d(u, W(x,y,\lambda) \leq \lambda\, d(u,x) + (I-\lambda)\, d(u, y), \text{ for all } u \in X.$$

X together with a convex structure is called a convex metric space. Clearly any convex subset of a normed space is a convex metric space with $W(x, y, \lambda) = \lambda x + (I-\lambda) y$.

DEFINITION 2. Let X be a convex metric space. A nonempty subset K of X is said to be convex if and only if $W(x, y, \lambda) \in K$ whenever $x, y \in K$ and $\lambda \in I$.

Takahashi has shown ([15]) that open and closed balls are convex, and that the arbitrary intersection of convex sets is convex too.

For arbitrary $A \subset X$ let

$$(1) \qquad \widetilde{W}(A) := \{W(x,y,\lambda) : x,y \in A, \lambda \in I\}.$$

It's easy to see that $\widetilde{W} : P(X) \rightarrow P(X)$ is a mapping with properties:

i) $A \subset \widetilde{W}(A)$, for $A \subset X$;

ii) $A \subset B$ implies $\widetilde{W}(A) \subset \widetilde{W}(B)$, for $A, B \in P(X)$

iii) $\widetilde{W}\left(A \cap B\right) \subset \widetilde{W}(A) \cap \widetilde{W}(B)$, for any $A, B \in P(X)$.

Using this notation we can say that K is convex iff $\widetilde{W}(K) \subset K$.

A few additional definitions will be needed too.

DEFINITION 3. A convex metric space X will be said to have Property (C) iff every bounded decreasing net of nonempty closed convex subsets of X has a nonempty intersection.

Remark. Every weakly compact convex subset of a Banach space has Property (C).

DEFINITION 4. Let X be a convex metric space and A be a nonempty, closed, convex, bounded set in X. For $x \in X$ we set

$$r_X(A) = \sup_{y \in A} d(x, y),$$

$$r(A) = \inf_{x \in A} r_X(A).$$

We then define $A_C = \{x \in A : r_X(A) = r(A)\}$ to be the centre of A. We denote the diameter of a subset A of X by

$$\delta(A) = \sup\{d(x, y) : x, y \in A\}.$$

DEFINITION 5. A point $x \in A$ is a diametral point of A iff $\sup_{y \in A} d(x,y) = \delta(A)$.

DEFINITION 6. A convex metric space X is said to have normal structure iff for each closed bounded convex subset A of X, which contains at least two points, there exists $x \in A$ which is not a diametral point for A.

Remark. it is clear that any compact convex metric space has normal structure.

4. RESULTS

Now, let us recall:

DEFINITION 7. The convex hull of a set A ($A \subset X$) is the intersection of all convex sets in X containing A and it is denoted by conv A.

It is obvious that if A is a convex set then $\widetilde{W}^n(A) = \widetilde{W}(\widetilde{W}(\ldots\widetilde{W}(A)\ldots) \subset A$ for any $n \in N$.

If we set :

$$A_n = \widetilde{W}^n(A), \qquad (A \subset X),$$

the sequence $\{A_n\}_{n\in N}$ is increasing so lim sup A_n exists and

$$\limsup A_n = \liminf A_n = \lim A_n = \bigcup_{n=1}^{\infty} A_n.$$

PROPOSITION 1. Let X be a convex metric space. Then

$$\text{(2)} \qquad \operatorname{conv} A = \lim A_n = \bigcup_{n=1}^{\infty} A_n, \qquad (A \subset X),$$

PROOF. If x, y$\in \bigcup_{n=1}^{\infty} A_n$, then there exists a positive integer n_o (say) such that x, y$\in A_{n_o}$. So, for $\lambda\in I$, $W(x, y, \lambda) \in A_{n_o+1} \subset \bigcup_{n=1}^{\infty} A_n$. Thus $\bigcup_{n=1}^{\infty} A_n$ is convex and contains A. Further for any convex set C containing A, $\widetilde{W}^n(A) \subset C$ for every $n \in N$, i.e., $\bigcup_{n=1}^{\infty} A_n \subset C$. So, $\bigcup_{n=1}^{\infty} A_n = \operatorname{conv} A$.

In the remainder of the paper (X, d) will denote a convex metric space.

PROPOSITION 2. For any subset *A* of (*X*, *d*)

$\delta(\operatorname{conv} A) = \delta(A)$.

PROOF. Since $A \subset \operatorname{conv} A$ then $\delta(A) \leq \delta(\operatorname{conv} A)$. Now, let *x* and *y* be in conv *A*. If $x, y\in A$ it is obvious that $d(x,y) \leq \delta(A)$, so let one of them, for instance *x*, be in conv $A \backslash A$ and the second one from *A*.

Since $x\in \operatorname{conv} A$ there exists $n_o\in N$ such that $x \in \widetilde{W}^{n_o}(A)$. But it means that there exist $x_1, x_2 \in \widetilde{W}^{n_o-1}(A)$, $\lambda_1 \in [0, 1]$ so that $x = W(x_1, x_2, \lambda_1)$ and then :

$$d(x, y) = d(W(x_1, x_2, \lambda_1), y) \leq \lambda_1 d(x_1, y) + (1 - \lambda_1) d(x_2, y).$$

By induction one can see that there exist subset $\{\tilde{x}_1\}_{i \in I} \subset A$ (I finite set) and $\{\alpha_i\}_{i \in I}$, $\alpha_i \geq 0, \Sigma \alpha_i = 1,$ such that

$$d(x, y) \leq \sum_{i \in I} \alpha_i d(\tilde{x}_i, y).$$

Since

$$d(\tilde{x}_i, y) \leq \delta(A) \qquad \text{for } i \in I$$

we prove that :

$$(3) \qquad d(x, y) \leq \delta(A).$$

Similarly one can prove that (3) is valid even in the case when $y \in$ conv A.

5. A FIXED POINT THEOREM

THEOREM. Let (X,d) be a metric space with continuous convex structure and let K be a closed convex bounded subset of (X, d) with normal structure and Property (C).

If $A: K \to K$ is a continuous mapping such that for $x,y \in K$

$$(4) \qquad d(Ax,Ay) \leq \max\{d(x,y), d(x, Ax), d(y,Ay)\, d(x, Ay), d(y, Ax)\}$$

then A has a fixed point.

PROOF, Let F be the family of all nonempty closed convex subset $F \subset K$ so that $A(F) \subset F$. Then F is nonempty since $K \in F$. Partilally order F by inclusion. Let $S = \{F_i\}_{i \in \Delta}$ be a decreasing chain in F.

From Property (C) it follows that $F_o = \bigcap_{i \in I} F_i \neq \phi$. So $F_0 \in F$.

Thus any chain in F has a greatest lower bound and by Zorn's Lemma there is a minimal member F in F. We claim that F is a singleton set. If not, then as shown by Takahashi [5] the center of F, denoted by F_c, is nonempt proper closed convex subset of F. It's easy to see that :

$$\delta(F_c) \le r(F) \le \delta(F).$$

Let us define a sequence :

$F_O = F_C$ and

$F_{k+1} = conv\,(F_k \cup A\,(F_k)),\ k = 0, 1,$ ....

Obviously $F_k \subset F_{k+1}$, (k = 0, 1, 2, ...) we shall prove by induction that :

(5) $\delta_k = \delta(F_k) \le r\,(F) = r$ for any $k \in N$.

For k=0 (5) is valid. Suppose that it is valid for k=0,1,2,...,....,m and let us prove that (5) is valid for $k = m+1$ too.

By definition of δ (F) for any sequence $\{\varepsilon_n\}$, $\varepsilon_n > o$ ($n \in N$), $\lim\limits_{n \to \infty} \varepsilon_n = 0$ there exist $\tilde{x}_n, \tilde{y}_n\, \varepsilon F_{m+1}$ so that

$$\delta_{m+1} - \epsilon_n \le d\,(\tilde{x}_n, \tilde{y}_n).$$

By Proposition 2 it is sufficient to analyse next three cases :

i) $\tilde{x}_n, \tilde{y}_n\, \varepsilon F_m$ $(n = 1, 2, ...)$,

ii) $\tilde{x}_n = x_n, \tilde{y}_n = Ay_n, (x_n, y_n, \varepsilon F_m, n = 1, 2, ..)$

iii) $\tilde{x}_n = Ax_n, \tilde{y}_n = Ay_n, (x_n, y_n, \varepsilon F_m, n = 1, 2, ..)$

At first case it is obvious that $\delta_{m+1} \le$ r. So let us see the second one. For any $x \in F_O$ we have that:

(6) $d\,(x, Ax)\ \le r.$

We shall suppose that (6) is valid for $x\, \varepsilon F_k$ $\{k = 0, 1, ...m-1\}$ and prove that it is valid for $k=m$.

For any $x\in F_m$, by Proposition 1,

$$x\in \widetilde{W}^{n_o}\left(F_{m-1}\cup A\left(F_{m-1}\right)\right)$$

for some $n_o \in N$.

Then

$$(7)\qquad d\,(x, Ax)\leq \sum_{j\in I_1} \gamma_j d\left(x_j, Ax\right)+\sum_{j\in I_2} \gamma_j d\left(Ax_j, Ax\right),$$

for $\quad x_j \in F_{m-1}, j\in I = I_1 \, U \, I_2$, (*I–finite set*) $I_1 \cap I_2 = \phi$

$$\sum_{j\in I} \gamma_j = 1,\ \gamma_j \geq 0 \text{ for } j\in I.$$

As in [7] it is sufficient to look only for the case when $\sum_{j\in I_1} \gamma_j \neq 0$. Further we have that :

$$d(x, Ax)\leq \sum_{j\in I_1} \gamma_j d\left(x_j, Ax\right)+\sum_{j\in I_2^{(1)}} \gamma_j d\left(x_j ,x\right)+\sum_{j\in I_2^{(2)}} \gamma_j d\left(x ,Ax\right)$$

$$+\sum_{j\in I_2^{(3)}} \gamma_j d\left(x_j , Ax_j\right)+\sum_{j\in I_2^{(4)}} \gamma_j d\left(x , Ax_j\right)+\sum_{j\in I_2^{(5)}} \gamma_j d\left(x_j ,Ax\right)$$

where we suppose
for $i\in I_2^{(1)}$ *that* $\quad d\,(Ax_j, Ax) \leq d\,(x_j, x)$;
for $i\in I_2^{(2)}$ that $\quad d\,(Ax_j, Ax) \leq d\,(x, Ax)$;
for $i\in I_2^{(3)}$ that $\quad d\,(Ax_j, Ax) \leq d\,(x_j, Ax_j)$;
for $i\in I_2^{(4)}$ that $\quad d\,(Ax_j, Ax) \leq d\,(x, Ax_j)$;
for $i\in 2(5)$ that $\quad d\,(Ax_j, Ax) \leq d\,(x_j, Ax)$;
and $I_2^{(j)} \cap I_2^{(k)} = \phi$ *for* $k\neq j$.

Now using the hypothesis one can see that

$$d(x, Ax)\leq \sum_{j\in I_1} \gamma_j\, d\left(x_j, Ax\right)+\ r\sum_{j\in I_2^{(1)}\cup I_2^{(3)}} \gamma_j + \sum_{j\in I_2^{(2)}} \gamma_j d\left(x, Ax\right)$$

$$+ \sum_{j\in I_2^{(4)}} \gamma d(x, Ax_j) + \sum_{j\in I_2^{(5)}} \gamma d(x_j, Ax).$$

Since, by induction similarly we have that

$$d(x, Ax_j) \le \sum_{k\in J_j^{(1)}} \beta_k d(\hat{x}_k, x_j) + \sum_{k\in J_j^{(2)}} \beta_k d(x_k, Ax_j)$$

$$+ \sum_{k\in J_j^{(3)}} \beta_k d(\hat{x}_k, A\hat{x}_k) + \sum_{k\in J_j^{(4)}} \beta_k d(x_j, A\hat{x}_k)$$

$$+ \sum_{k\in J_j^{(5)}} \beta_k d(\hat{x}_k, A\hat{x}_j)$$

$$\text{for} \quad \hat{x}_k \in F_{m-1}, k\left(\in J_j = \bigcup_{i=1}^{5} J_j^{(i)}, \sum_{k\in J_j} \beta_k = 1, \beta_k \ge 0, k\in J_j, \sum_{k\in J_j^{(1)}} \beta_k \ne 0\right).$$

So

$d(x, Ax_j) \le r$ and

$$d(x, Ax) \le \sum_{j\in I_1} \gamma_j d(x_j, Ax) + \left(\sum_{j\in I_2^{(1)}} + \sum_{j\in I_2^{(3)}} + \sum_{j\in I_2^{(4)}}\right) g$$

$$(8) \quad + \sum_{j\in I_2^{(2)}} \gamma_j d(x, Ax) + \sum_{j\in I_2^{(5)}} \gamma_j d(x_j, Ax)$$

After not more than n_0- steps we shall have that :

$$d(x, Ax) \le \sum_{j\in I^*} \gamma_j^* d(v_j, Ax) + \gamma_0^* r, \quad \text{for } \gamma_i^* \ge 0 \;\; i \in \{0\} \cup I^*$$

$\gamma_0^* + \sum_{j\in I^*} \gamma_j^* = 1$ *and* $v_j \in F_0, j\in I^*$. Since F_0 is centre we have that

$d(v_j, Ax) \le r$

which implies that :

$d(x, Ax) \le r$ for all $x\in F_m$.

Similarly one can prove that $d(x, Ay) \le r$ for all $x, y \in F_m$

so in the second case we have

$$\delta_{m+1} - \varepsilon_n \le d(\tilde{x}_n, \tilde{y}_n) = d(x_n, Ay_n) \le r, n \in N$$

and consequently

$$\delta_{m+1} \le r.$$

Using (4) it is easy to prove this inequality in the case iii). So for any $m \in N$

$$\delta_m \le r.$$

Let us define $F_\infty := \bigcup_{k=0}^{\infty} F_k$

F_0 is nonemty set. So F_∞ is nonempty too.

Since $\delta(F_\infty) < r < \delta(F)$, F_∞ is closed proper subset of F. Moreover, W is continuous and this guarantees that the closure of convex set is convex. Further, mapping A is continuous so:

$$A(F_\infty) \subset F_\infty$$

Therefore $F_\infty \subset F$, a contradiction to the minimality of F. Hence F consists of a single element which is a fixed point for A.

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THERMAL STRESS IN A SEMI-INFINITE ROD OF NONHOMOGENEOUS MATERIAL

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ABSTRACT

Distribution of thermal stress in a semi-infinite elastic rod of nonhomogeneous material has been found. The density and the Young's modulus of the material are supposed to be functions of position. A constant temperature has been maintained at the end of the rod. Distribution of stress has been shown graphically for particular values of the parameters involved.

Key words : Thermal Stress, Young's modulus, Nonhomogeneous material, Laplace transform.

INTRODUCTION

The problem of determining thermal stresses in a elastic body has been gaining increasing importance because there appear many practical problems where mechanical structures are subjected to high temperature and pressure. Barrekette [1], Lal [3] and Nowinski [4] considered statical problems. But due to complexity involved in the governing differential equations it becomes quite difficult to solve thermoelastic problems when time appears as a variable. The problem of determining stress in rods is usually based on a one-dimensional stress-strain law and the hypothesis that plane sections of the rod remain plane after heating. The dynamical thermoelastic problem had been solved by Roychaudhuri [6] where he found the distribution of thermal stress in a thin rod of finite length one end of which was maintained at a constant temperature, the material of the rod being homogeneous. As regards nonhomogeneous problems, the governing differential equations become little more complicated and as a result sometimes simplifying assumptions are adopted in order to avoid such complications. The technique of Laplace transform is used, but due to natural difficulties in inversion, asymptotic expansion is followed to get solution of the problem valid for small time or limiting values of parameters are assumed. Latter type of non-homogeneous problem has been considered by Pal [5]. In the present investigation an attempt has been made to find thermoelastic stress distribution in a semi-infinite rod the end of which has been maintained at a constant temperature. The material of the rod has been assumed nonhomogeneous in the sense that both young's modulus E and the density ρ of the material are functions of position.

FORMULATION OF THE PROBLEM

Choosing x-axis along the axis of the rod let the position of the rod be given by $x \geq 0$. The differential equation of motion is

$$\frac{\partial \sigma}{\partial x} = \rho \frac{\partial^2 u}{\partial t^2} \quad \ldots (1)$$

where $\rho = \rho(x)$ is the density of the material and $u = u(x,t)$ is the longitudinal displacement of the centroid of the cross-section, which in the equilibrium state is at a distance x from the origin. The $\sigma = \sigma(x,t)$ is given by

$$\sigma = E\left(\frac{\partial u}{\partial x} - \alpha T\right) \quad \ldots (2)$$

where $E = E(x)$ is the Young's modulus, $T = T(x,t)$ is the temperature at time t at α distance x from the end and α is the coefficient of linear expansion of the material of the rod.

Let the nonhomogeneity of the material be characterised by

$$E = E_o e^{-az} \qquad \dots (3)$$

$$\rho = \rho_o e^{-az}, \; a \neq o$$

where $z = x/L$ ... (4)
and L has the dimension of length.

In view of (3), substitution of (2) into (1) leads to

$$\frac{\partial^2 u}{\partial z^2} - a\frac{\partial u}{\partial z} - \frac{\partial^2 u}{\partial \tau^2} = \alpha L \left(\frac{\partial T}{\partial z} - aT\right) \qquad \dots (5)$$

where $\tau = \frac{ct}{L}$, $c^2 = E_o/\rho_o$. ... (6)

The differential equation for the determination of temperature $T(x,t)$ at any point x at time t is

$$\frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial x^2}, \; x > 0, t > 0 \qquad \dots (7)$$

The quantity k generally depends upon the thermal conductivity, specific heat and the density of the material. For the sake of simplicity in the solution of (7) we assume that k is a constant.

Using (4) and (6), (7) becomes

$$\frac{\partial T}{\partial t} = h\frac{\partial^2 T}{\partial z^2} \qquad \dots (8)$$

Where $h = k/cL$.

The initial and boundary conditions of the problem are

$$u(z, 0) = 0 = \frac{\partial}{\partial \tau} u(z, 0), \; z > 0 \qquad \dots (9)$$

$$T(z, 0) = 0 \qquad \ldots (10)$$

$$\frac{\partial u}{\partial z} = \alpha LT, z = 0, \tau > 0 \qquad \ldots (11)$$

$$T(0, \tau) = T_o, \tau > 0 \qquad \ldots (12)$$

In addition to the above conditions we must have the regularity condition that the stress σ, displacement u and temperature T are bounded for all z and τ.

SOLUTION OF THE PROBLEM

Let us denote the Laplace transform with respect to τ, of any function $g(z,\tau)$ by $\overline{g}\,(z,p)$ where p is the transform parameter.

With this notation, Laplace transforms of (5) and (8), taking into account of initial conditions (9) and (10), yield

$$\frac{\partial^2 \overline{u}}{\partial z^2} - a\frac{\partial \overline{u}}{\partial z} - p^2 \overline{u} = \alpha L\left(\frac{\partial \overline{T}}{\partial z} - a\overline{T}\right) \qquad \ldots (13)$$

and

$$\frac{\partial^2 \overline{T}}{dz^2} = \frac{p}{h}\,\overline{T} \qquad \ldots (14)$$

Laplace transform of boundary conditions (11) and (12) yield

$$\frac{\partial \overline{u}}{\partial z} = \alpha L\overline{T}, z = 0 \qquad \ldots (15)$$

$$\overline{T}\,(O, p) = \frac{T_0}{p} \qquad \ldots (16)$$

Solution of (14) with boundary condition (16) is given by Spiegel [7],

$$\overline{T}\,(z, p) = \frac{T_o}{p} e^{-\sqrt{\frac{p}{h}}\,z} \qquad \ldots (17)$$

In view of (17), differential equation (13) may now be solved for $\bar{u}$ as

$$\bar{u}\,(z,p\,) = A\,e^{\lambda z} - \frac{\alpha L T_o}{p}\,\frac{\left(\sqrt{\frac{p}{h}}+a\right)e^{-\sqrt{\frac{p}{h}}\,z}}{a\sqrt{\frac{p}{h}}+\frac{p}{h}-p^2} \qquad \text{... (18)}$$

where the regularity condition $\bar{u}\,(\infty\,,p) = 0$ has been used. The quantity λ is given by

$$\lambda = \mu - \sqrt{p^2+\mu^2} \qquad \text{... (19)}$$

where $\mu = a/2$.

The constant A may be determined by using (15) and (16). Finally, we obtain $\bar{\sigma}(z,p)$ as

$$\bar{\sigma}(z,p) = \alpha\,E_o E\,e^{-\alpha z}\left[e^{-z\sqrt{p/h}} - e^{\lambda z}\right]\frac{p}{\frac{p}{h}+a\sqrt{\frac{p}{h}}-p^2} \qquad \text{... (20)}$$

If β_1, β_2, β_3, are the roots of the cubic

$$y^3 - b^2 y - ab^2 = 0 \qquad \text{... (21)}$$

where $b = 1/h$ and $y = \sqrt{p/h}$, then (20) may be written as

$$\bar{\sigma}\,(z,p\,) = \alpha\,E_o T_o\,e^{-\alpha z}\left[e^{\lambda z}\sum_{i=1}^{3}\frac{L_i}{\sqrt{p}\left(\sqrt{p}+\beta_i\,\sqrt{h}\right)} - \frac{e^{-z\sqrt{p/h}}}{h}\sum_{i=1}^{3}\frac{M_i}{p\left(\sqrt{p}-\beta_i\,\sqrt{h}\right)}\,\frac{1}{p}\,e^{-z\sqrt{p/h}}\right] \qquad \text{... (22)}$$

where

$$L_i = \frac{\beta_i^{\,2}}{\left(\beta_j-\beta_i\right)\left(\beta_k-\beta_i\right)}$$

$$M_i = \frac{a-\beta_i}{(\beta_j-\beta_i)(\beta_k-\beta_i)}, \; i \neq j \neq k, \; i, j, k = 1, 2, 3$$

Equation (22) may now be used to find the stress σ (z, τ) at any point z at time τ by applying inverse Laplace transform [cf. Erdelyi [2]]

$$\sigma(z,\tau) = \alpha E_o T_o e^{-az/2}\left[G(\tau-z) H(\tau-z) + \int_z^{\tau} f(u,z)\, G(\tau-u)\, du \right]$$

$$+ \alpha E_o T_o \, e^{-az}\left[g(\tau,z) - \text{erfc}\left(z/2\sqrt{h\tau}\right)\right] \qquad \ldots (23)$$

where

$$f(\tau,z) = -\frac{\mu z}{\sqrt{\tau^2-z^2}} J_1\left(\mu\sqrt{\tau^2-z^2}\right) H(\tau-z)$$

$$G(\tau) = \sum_{i=1}^{3} L_i \, e^{\beta_i^2 h\tau} \text{erfc}\left(\beta_i \sqrt{h\tau}\right)$$

$$g(\tau,z) = \sum_{i=1}^{3} \frac{M_i}{h^{3/2}\beta_i}\left[\text{erfc}\left(z/2\sqrt{h\tau}\right) - e^{\beta_i z + \beta_i^2 h\tau}\, \text{erfc}\left(\frac{z}{2\sqrt{h\tau}} + \beta_i\sqrt{h\tau}\right)\right]$$

$$\text{erfc}\,(z) = \frac{2}{\sqrt{\pi}} \int_z^{\infty} e^{-t^2} dt$$

and H (τ) is Heaviside's step function.

NUMERICAL RESULTS

To get an idea about the effect of nonhomogeneity as stipulated in (3) on the stress, variations of stress with position and time as given by (23) has been numerically computed. Although the roots $\beta_1, \beta_2, \beta_3$ of the cubic in (21) may be found by any standard method, the restriction $b = 3\sqrt{3a}$ makes the procedure quite simple. In our numerical evaluation we have used this condition and get $\beta_1 = -5.63816\,a$, $\beta_2 = 4.59625\,a$, $\beta_3 = 1.04190\,a$.

In figures 1 and 2 we have shown the variations of

$$p = \frac{\sigma}{\alpha E_o T_o}$$

With τ over the range 0 to 6.0 for τ. In Fig. 1 the parameter a in (3) has been fixed at 0.2 and the variations of stress with positions have been shown. It is observed from the graph that at a particular position stresses diminish with time while within the range of τ considered the magnitude of stress at a particular time for points on the right of the rod is greater than that for the points in the left. From the nature of the graph it is apparent that for sufficiently large values of z he stress becomes negligible, which is in conformity with the regularity condition. In Fig. 2 we have plotted the variations of P with time keeping z fixed at 2. It is observed that with the increase of a the stress diminishes, while the behaviour of the stress remaining the same.

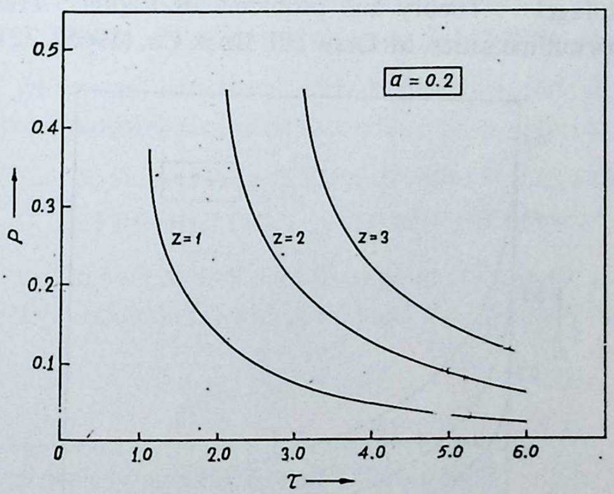

Fig.1 Variation of stress with time (for fixed a)

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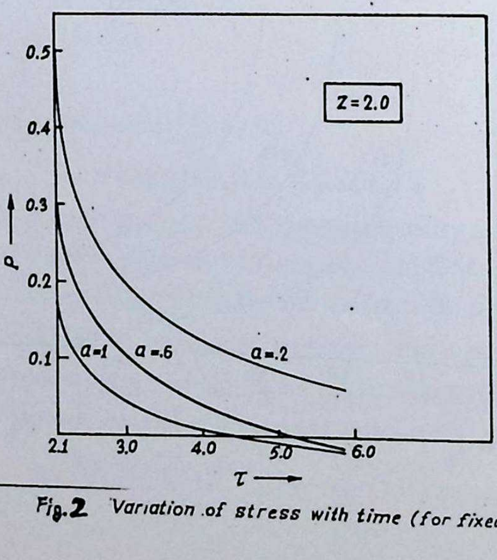

Fig.2 Variation of stress with time (for fixed z)

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COINCIDENCES AND FIXED POINTS OF MEIR-KEELER TYPE CONTRACTIVE MAPPINGS ON MENGER SPACES

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ABSTRACT

In this paper the notion of Meir-Keeler type contractive triplet of mappings on a Menger space is introduced and some coincidence and fixed point theorems for such mappings are established. The present results generalize a number of well-known results in fixed point theory.

Key words and phrases : Meir-Keeler type contractive triplet, Menger space, probabilistic metric space, fixed point, coincidence point.

AMS (MOS) Subject Classifications (1980) : 54H25, 54E99

INTRODUCTION AND DEFINITIONS

The well-known Meir-Keeler fixed point theorem [8] proved under the contractive condition (1) with A = M and T = I (Identity mapping) has been applied, extended and studied, among others by Chung [2] Ćiric´ [3] Maiti-pal [6], Matkowski-Wegrzyk [7], Park [9] Park-Bae [10], Park-Moon [11], Park-Rhoades [12], Rao-Rao [13], Rhoades [14], Shisheng [16], Singh [17], Singh-Virendra [20] and Yen-Chung [21]. For fixed point theorems for three mappings on a metric space satisfying Meir-Keeler type contractive conditions, refer to Park-Moon [11] and Rhoades [14]. However, fixed point theorems of these authors and several others, except of course [13] and [17], require continuity condition on the mapping(s) under consideration.

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Recent coincidence theorems (on metric spaces) of Park [9] and Singh [17] extend and unify a coincidence theorem of Goebel [4], Meir-Keeler fixed point theorem [8] and a few fixed point theorems for contractive mappings.

In this paper we introduce Meir-Keeler type contractive conditions in Menger spaces and establish coincidence and fixed point theorems for three mappings, wherein either we use no continuity conditions or a very mild continuity condition is used.

In all that follows, A stands for an arbitrary nonempty set, (M,d) for a metric space, X for a Menger space (X, f, t) where t is continuous and satisfies $t(x, x) \geq x, x \in [0,1]$. By $C(P,T)$ we denote the set of coincidence points of mappings P and T, i.e., $C(P,T) = \{z \in A : Pz = Tz\}$.

Definition 1. A probabilistic metric space (PM-space) is an ordered pair (X, f) consisting of a nonempty set X and a mapping, f form $X \times X$ to L the collection of distribution functions. We shall represent the value of f at $(u,v) \in X \times X$ by $F(u, v)$. The functions $F(u, v)$ are assumed to satisfy the following conditions :

(a) $F(u,v;x) = 1$ for all $x > 0$, iff $u = v$;

(b) $F(u,v;0) = 0$ for all $u,v \in X$;

(c) $F(u,v) = F(v,u)$ for all $u, v \in x$;

(d) if $F(u, v; x) = 1$ and $F(v, w; y) = 1$ then

$F(u, w; x+y) = 1.$

A Menger space is a triplet (X, f, t) where (X, f) is a PM-space and t-norm (or T-norm [15]) t is such that (d) is replaced by

(d') $F(u,w; x+y) \geq t\,(F(u,v; x), F\,(v,w,y))$

for all $x, y \geq 0$. For the details of topological preliminaries, refer to schweizer and Sklar [15].

Let P, Q, T be mappings on A with values in M or X as per situation. Consider the following conditions :

(1) For a given $\varepsilon > 0$, there exists a $\delta = \delta(\varepsilon) > 0$ Such that for $u, v \in A$, $\varepsilon \leq d\,(Tu, Tv) < \varepsilon + \delta +$ implies $d\,(Pu, Pv) < \varepsilon.$

(2) For a given $\varepsilon > 0$, there exists a $\delta = \delta(\varepsilon) > 0$ such that for $u, v \in A$, $\varepsilon \leq \max\{d\,(Tu, Tv), d\,(Tu, Pu), d\,(Tv, Qv), 1/2\,[d(Tu, Qv) + d\,(Tv, Pu)]\}$

$< \varepsilon + \delta$ implies $d(Pu, Qv) < \varepsilon.$

The condition (1) has been used to generalize Goebel's coincidence theorm [4] by Park [9], while Park-Bae [10], using the condition (1) with A = M and PT = TP, have unified the fixed point theorems of Jungck [5] and Meir-Keeler [8]. A recent fixed point theorem of Park-Moon [11, Th.3.1], proved under the condition (2) with A = M, T continuous and commuting with each of P and Q extends and unifies a number of Meir-Keeler type fixed point theorems. Park-Moon [11] have also given a nice comparison of Meir-Keeler type contractive mappings. Note that (2) includes (1). Now, akin to (2) we introduce a Meir-Keeler type contractive condition for three mappings in a Menger space. Our results proved under this condition extend and include fully a number of results, among others, from [3-6], [8-10], [18], [19], and partially from [11-13], [17] and [20].

Let min $\{F(Tu, Tv; x), F(Tu, Pu; x), F\,(Tv, Qv; x)$

$F(Tu, Qv; 2x), F(Tv, Pu; 2x)\} = K(u, v; x)$ *for* u, v *in* A *and* $x > 0$.

Definition 2. Three selfmappings P, Q, T on X will be called a Meir-Keeler type contractive triplet if given $\varepsilon > 0$ there exists a $\delta = \delta(\varepsilon) > 0$ such that

(3) $0 = K(u, v; \varepsilon) < K(u, v; \varepsilon + \delta) = 1$ implies

$F(Pu, Qv; \varepsilon) = 1$.

Definition 3. Let P, Q, T be selfmappings on X If there exists a point u_0 in X and a sequence $\{u_n\}$ in X such that

$$Tu_{2n+1} = Pu_{2n}, Tu_{2n+2} = Qu_{2n+1}, n = 0,1,2,.....$$

then the space X is called $(P, Q; T(u_0))$ - orbitally complete with respect to μ_0 or simply $(P,Q; T)$ - orbitally complete if the closue of $\{T_{u_n} : n = 1,2,.......\}$ is complete.

Definition 4. The mapping T is called $(P,Q,; T(u_0))$ - orbitally continuous if the restriction of T on the closure of $\{Tu_n : n = 1,2,......\}$ is continuous.

Definition 5. Two selfmapings P and T on X will be called weakly commuting if,

$F(PTu, TPu; x) \geq F(Tu, Pu; x)$ *for all* $u \in X$ *and* $x > 0$,

RESULTS

The following lemma is motivated by $S.- S.$ Chang [1].

Lemma. Let $\{y_n\}$ be a sequence in X such that $\lim_n F(y_n, y_{n+1}; \varepsilon) = 1$ for any $\varepsilon > 0$. If $\{Y_n\}$ is not Cauchy then there exist $\varepsilon_0 > 0$ and two sequences of positive integers, $\{m_i\}$ and $\{n_i\}$, such that

(a) $m_i > n_i + 1, i \to \infty, n_i \to \infty;$

(b) $F(y_{m_i}, y_{n_i}; \varepsilon_0) \leq 1 - \lambda, \quad 1 = 1, 2, ...,$

(c) $F(y_{m_i-1}, y_{n_i}; \varepsilon_0) > 1-\lambda, \quad i = 1, 2, \ldots$

Proof. Since $\lim_n F(y_n, y_{n+1}; \varepsilon) = 1$, for any $\varepsilon_0, \lambda > 0$, there exists a positive integer N such that $F(y_n, y_{n+1}; \varepsilon_0) > 1-\lambda$ for $n \ge N$. As the sequence is not cauchy, we have for $\varepsilon_0, \lambda > 0$,

$$F(y_{n_1}, y_{p_1}; \varepsilon_0) \le 1-\lambda, p_1 > n_1 > N_1 > N.$$

Let $S = \{p \in I^+ : F(Y_{n_1}, Y_p; \varepsilon_0) \le 1-\lambda,\ p > n_1\}$, I^+ being the set of positive integers. Then S is clearly nonempty. Let $p_0 \in S$ such that $p_0 \le n_1 + 1$. Then $n_1 + 1 \ge p_0 > n_1 > N$, *i.e.*, $p_0 = n_1 + 1$. Thus,

$F(y_{n_1}, y_{p_0}; \varepsilon_0) = F(y_{n_1}, y_{n_1+1}; \varepsilon_0) > 1-\lambda$, a contradiction, implying $p > n_1+1, p \in S$. Now we can find a minimal integer $m_1 \in A$ such that $m_1 < n_1 + 1$, $F(y_{n_i}, y_{m_i}; \varepsilon_0) \le 1$ [illegible] $F(y_{n_i}, y_{m_i}; \varepsilon_0) < 1 - \lambda$. Repeated use of these arguments gives the desired conclusions.

पुस्तकालय
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Theorem 1. Let P, Q, T be a Meir-Keeler type contractive triplet on A with values in X. If there exists a point u_0 in A such that X is $(P, Q; T(u_0))$ – orbitally complete then P, Q, T have a coincidence, i.e., there exists a point z in A such that $Pz = Qz = Tz$. Further, if $A = X$ and for all $u, v \in X, x > 0$, either

(i) $F(Qu, Tv; x) \ge \min\{F(v, Qu; x), F(v, Tu; x)\}$

or

(ii) $F(Pu, Tv; x) \ge \min\{F(v, Tu; x), F(v, Pu; x)\}$ then

P, Q, T have a unique common fixed point and $\{Tu_n\}$ converges to the fixed point.

Proof. Condition (3) is equivalent to

$$(4) \quad F(Pu, Qv; x) > \min\{F(Tu, Tv; x), F(Tu, Pu; x), F(Tv, Qv; x), F(Tu, Qv; 2x), F(Tv, Pu; 2x)\}.$$

Two cases arise.

Case I. suppose $Tu_n = Tu_{n+1}$ and $Tu_{n+1} \neq Tu_{n+2}$ for some odd integer n. By (4)

$$F(Tu_{n+2}, Tu_{n+1}; x)$$

$$= F(Pu_{n+1}, Qu_n; x)$$

$$> \min \{F(Tu_{n+1}, Tu_n; x), F(Tu_{n+1}, Tu_{n+2}; x), F(Tu_n, Tu_{n+1}; x), F(Tu_{n+1}, Tu_{n+1}; 2x), F(Tu_n, Tu_{n+2}; 2x)\}$$

$= F(Tu_{n+1}, Tu_{n+2}; x)$, since by (d')

$$F(Tu_n, Tu_{n+2}; 2x) \geq \min \{F(Tu_n, Tu_{n+1}; x), F(Tu_{n+1}, Tu_{n+2}; x)\}.$$

But this is a contradiction.

Hence $Tu_{n+1} = Tu_{n+2}$, *i.e.*, in general

$$Tu_{n+i} = Tu_n,\ i = 1, 2, \ldots$$

Taking $u_n = z$, we have

(5) $Tu_n = Tu_{n+1} = Qu_n$, *i.e.*, $Tz = Qz$.

Let $Pz \neq Qz$, then from (4),

$F(Pz, Qz; x) > F(Pz, Qz; x)$, a contradiction.

Thus, $Pz = Qz$. Hence, from (5),

$$Pz = Qz = Tz.$$

The case when n is even may be dealt with similarily.

Case II. Suppose $Tu_n \neq Tu_{n+1}$ for all integers n. The application of (4) gives that $\{F(Tu_n, Tu_{n+1}; x)$ is an increasing sequence converging to a number $0 \leq r \leq 1$. Let $r < 1$.

From this we infer that for every $s \in (0, r)$ there exists a positive integer N such that $r - s \leq F(Tu_n, Tu_{n+1}; x) < r$ for all $n \geq N$, since the convergence is monotone from the left.

Let $2k \geq N$, then by (4)

$$F(Tu_{2k+1}, Tu_{2k+2}; x) > \min\{F(Tu_{2k}, Tu_{2k+1}; x), F(Tu_{2k}, Pu_{2k}; x),$$
$$F(Tu_{2k+1}, Qu_{2k+1}; x), F(Tu_{2k}, Qu_{2k+1}; 2x),$$
$$F(Tu_{2k+1}, Pu_{2k}; 2x)\} = F(Tu_{2k}, Tu_{2k+1}; x).$$

Thus $F(Tu_{2k+1}, Tu_{2k}+2; x) > r$, a contradiction implying $r = 1$.

To prove that the sequence $\{Tu_n\}$ is a Cauchy sequence, suppose the contrary to be true. Then, by the Lemma, there exists $\varepsilon_0 > 0$ and two sequences of positive integers,

$\{m_i\}$ and $\{n_i\}$, such that

(a) $m_i > n_i + 1, n_i \to \infty, i \to \infty$;

(b) $F(Tu_{m_i}, Tu_{n_i}; \varepsilon_0/2) \leq 1 - \lambda, i = 1, 2, \ldots$;

(c) $F(Tu_{m_i-1}, Tu_{n_i}; \varepsilon_0/2) > 1 - \lambda, i = 1, 2, \ldots$.

Thus using (b), (c) and passing on to the limits we have

$$F(Tu_{m_i}, Tu_{n_i}, e_0) \geq \min\{F(Tu_{m_i}, Tu_{m_i}-1; \varepsilon_0/2),$$
$$F(Tu_{m_i-1}, Tu_{n_i}; \varepsilon_0/2)\} > \min\{1, 1 - \lambda\} = 1 - \lambda,$$

a contradiction.

Therefore $\{Tu_n\}$ is a Cauchy sequence. Since $\times$ is $(P, Q, T(u_0))$ – orbitally complete, there exists a z in $\times$ such that $Tu_n \to z$.

Now if (i) holds, then

$F(Qu_{2n+1}, Tz; x) \geq \min\{F(z, Qu_{2n+1}; x), F(z, Tu_{2n+1}; x)\}$, implying $Tz = z$.

And if (ii) holds then also $Tz = z$.

Also, since $Tu_m \neq Tu_{m+1}$, we get by (4),

$F(z, Qz; x) > F(z, Qz; x)$, giving $z = Qz$ which in trun, again with the application of (4), gives $z = Pz$.

Thus $\quad Pz = Qz = Tz = z$.

With suitable choice of P, Q, T in (3) we get probabilistic analogues of various well-known coincidence and fixed point theorems in metric spaces, some of them being due to Goebel [4], Meir-Keeler [8], Park [9], Park-Bae [10], Rao-Rao [13], Singh [17] and Singh-Virendra [20].

Theorem 2. Let $P, Q, T : X \to X$ be a Meir - Keeler type contractive triplet. If there exists a point u_0 in X such that X is $(P, Q; T(u_0))$ – orbitally complete and,

either (i) $PTz = TPz, z \in C(P, T)$;

or (ii) $QTz = TQz, z \in C(Q, T)$, then P, Q, T, have a unique common fixed point and $\{Tu_n\}$ converges to the fixed point.

Proof. The uniquene of the fixed point of P, Q, T if it exists, is ensured by (4) with $A = X$. So, in view of Case I of the proof of Theorem 1, it is sufficient to show that P, Q, T have a common fixed point $Tz(=Pz=Qz)$. Without loss of generality we may assume that P and T commute at $z \in C(P, T)$.

Suppose (i) holds. Then

(6) $\quad PTz = TPz, = TTz$, as $Tz = Pz$.

Now taking $TTz \neq Tz$, we have by (4),

$F(TTz, Tz; x) = F(PTz, Qz; x) > F(TTz, Tz_i\ x)$, a contradiction.

Thus, in view of (6)

$TTz = Tz = PTz$.

Clearly $QTz = Tz$, since otherwise it leads to a contradiction. Hence $Tz(=Pz=Qz)$ is a common fixed point of P, Q, T. Hence $Tz\ (=Pz=Qz)$ is a common fixed point of P, Q, T.

Remark: It is interesting to note that every pair of mappings (P, T) commuting weakly on $C(P,T)$ is commuting on $C(P,T)$. Thus weak commutativity of P and T at a coincidence point z of P and T is equivalent to the commutativity of P and T at z. Therefore Theorem 2 holds even if commutativity of P and T (resp. Q and T) is replaced by weak commutativity of P and T (resp. Q and T) Thus the proof of the following theorem is obvious.

Theorem 3 . Let $P, Q, T, : X \to X$ be a Meir-Keeler type contractive triplet. If there exists a point u_0 in X such that X is $(P, Q,; T(u_0))$ – orbitally complete and either

(i) P and T are weakly commuting and $(P,Q;T(u_0)$ –orbitally continuous;

or

(ii) Q and T are weakly commuting and $(P, Q; T(u_0))$ –orbitally continuous, then P, Q, T have a unique common fixed point p (say) in X and for some $u_0 \in X$, the sequence $\{Tu_n\}$ converges to a point z in X such that $Tz = p$.

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LOGISTIC GROWTH UNDER RANDOM DRIVING FORCES

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ABSTRACT

The stochastic behaviour of non-equilibrium fluctuation in the logistic model of growth under random driving forces has been studied in the light of the 'fluctuation-dissipation' theorem of statistical mechanics.

Key words : Logistic growth equation, Carrying capacity, Non-equilibrium fluctuation, White-noise, Fluctuation-dissipation theorem.

AMS (MOS) subject classification (1980) : 92A17

1. INTRODUCTION

Logistic growth equation plays an unique place as the first non-linear equation for population growth. It finds wide application in different branches of biology. It is also the basic equation of non-linear autocatalytic reactions which is fundamental in understanding many phenomena of life processes [7]. A lot of works was done on this equation covering various aspect of population, but there are others to be studied from a different consideration.

In this paper we shall study the stochastic behaviour of non-equilibrium fluctuation in the logistic growth of a biological system under the influence of random driving force. The randomness appeared here is assumed to be internal, that is, due to the random interactions among the individuals of a single species [3]. Two cases namely, the state with constant carrying capacity and the other with time-varying capacity have been considered.

2. VERHULST GROWTH UNDER RANDOM DRIVING FORCE

A. Constant Carrying Capacity

The Verhulst (or Logistic) equation of growth of a single species population is given by

$$\frac{dN}{dt} = \alpha N - \beta N^2 = \alpha N \left(1 - \frac{N}{K}\right) \qquad \text{... (1)}$$

where $N(t)$ is the population size at any time t, α is the intrinsic rate constant and $K = \alpha/\beta$ is the carrying capacity of the environment. Let us first assume that K or β is constant. This type of equation occurs in different cases of biophysical system [7]. The stochastic extension of the equation (1) is given by

$$\frac{dN}{dt} = \alpha N - \beta N^2 + \varepsilon(t) \qquad \text{... (2)}$$

where $\varepsilon(t)$ is the random perturbation term which is due to the overall effect of the numerous microscopic, unknown or partially known internal fluctuation of the state variable N. We have called the randomness of perturbation to be internal in the sense that it is due to the internal effect of mutual interactions of the individuals of the same species without any consideration of the randomness of the environmental parameters. The random perturbation term $\varepsilon(t)$ is assumed to be a white noise satisfying the condition ·

$$\langle \varepsilon(t) \rangle = 0, \ \langle \varepsilon(t_1)\, \varepsilon(t_2) \rangle = 2D_0\, \delta\,(t_1 - t_2) \qquad \text{... (3)}$$

where < > represents the average over the ensemble of the stochastic process. D_0 is the diffusion coefficient. The equation (2) is a non-linear stochastic differential equation of non-linear Langevin type and there are some difficulties in solving such problem [6]. We shall, however, resort to the linearization method to study the characteristic stochastic behaviour of the system. The equation (2) is linearized as follows :

The equation for the average $\langle N \rangle$ is given by

$$\frac{d\langle N \rangle}{dt} = \alpha \langle N \rangle - \gamma \langle N^2 \rangle \qquad \text{...(4)}$$

or, approximately by

$$\frac{d\langle N \rangle}{dt} = \alpha \langle N \rangle - \gamma \langle N \rangle^2 \qquad \text{... (5)}$$

Where we have used the relation $\langle N^2 \rangle = \langle N \rangle^2$ as a first approximation.

Let us put $N(t) = \langle N \rangle + N_0(t)$ ... (6)

in equation (2) and subtracting (4) from it, we get the linear equation

$$\frac{dN_0}{dt} = \left(\alpha - 2\beta \langle N \rangle\right) N_0 + \varepsilon(t) \qquad \text{... (7)}$$

Where we have neglected second order terms of $N_0(t)$.

If $\sigma^2(t) = \left\langle (N - \langle N \rangle)^2 \right\rangle = \left\langle N_0^2 \right\rangle$

the $\sigma^2(t)$ satisfies the ordinary differential equation [2]

$$\frac{d}{dt}\sigma^2(t) = 2\left(\alpha - \beta \langle N \rangle\right)\sigma^2(t) + 2D_0 \qquad \text{... (8)}$$

For $\alpha > 0$ and for large t, $\langle N \rangle \to 0$ so that the equation (8) takes the form

$$\frac{d\left\langle N_0^2(t)\right\rangle}{dt} = 2\alpha\left\langle N_0^2(t)\right\rangle + 2D_0 \quad \text{... (9)}$$

Then $\left\langle N_0^2(t)\right\rangle\Big|_{t \to \text{lage}} = \dfrac{D_0}{\alpha}$ ... (10)

B. Quasi-steady State : Fluctuation

Let us now consider the situation of the quasi-steady state which corresponds to the state very near to the steady non-equilibrium state. Such a state is characterized by the slow variation of the drift coefficient $A(t)\left(\alpha - \beta\ \langle N\rangle\right)$ and the diffusion coefficient D_0 which is assumed here to be constant. In addition, the addmittance functionn $Z(i\omega, t)$ relating $N_0(t)$ and $e(t)$ of the stochastic equation (7) must be slowly varying function of time. Under these assumptions, the mean square fluctuation of the population size $N(t)$ is given by [1]

$$\left\langle N_0^2(t)\right\rangle = \frac{D_0}{\pi}\int_{-\infty}^{\infty}\left|\frac{A(t)}{\left(A(t)+i\omega\right)} + \frac{A(t)}{\left(A(t)+i\omega\right)^3}\right|^2 d\omega \quad \text{... (11)}$$

The above formula is a generalization of the famous Nyguist's relation for quasi-stationary state [1]. For the slowly varying condition of the drift coefficient $A(t) = \left(\alpha - \beta\ \langle N\rangle\right)$. We must have

$$\left|\frac{\dot{A}(t)}{A(t)}\right| << 1 \quad \text{... (12)}$$

or, $\left|\beta\ \langle N\rangle\right| << 1$ ... (13)

Under this condition let us now find the value of $\left\langle N_0^2(t)\right\rangle$. Now (11) can be reduced to the form

$$\left\langle N_0^2(t)\right\rangle = \frac{D_0}{\pi}\int_{-\infty}^{\infty}\left[\frac{1}{\left(A^2(t)+\omega^2\right)} - \frac{2\dot{A}\cdot(t)}{\left(A^2(t)+\omega^2\right)^2} + \frac{4\dot{A}(t)A^2(t)+\dot{A}^2(t)}{\left(A^2(t)+\omega^2\right)^3}\right]d\omega \qquad \ldots (14)$$

Since $A(t)=\alpha-\beta\left\langle N\right\rangle,\ \dot{A}(t)=-\beta\left\langle N\right\rangle A(t)$

So

$$\left\langle N_0^2(t)\right\rangle = \frac{D_0}{\pi}\left[\int_{-\infty}^{\infty}\frac{d\omega}{A^2(t)+\omega^2} + 2\beta A(t)\langle N\rangle\int_{-\infty}^{\infty}\frac{d\omega}{\left(A^2(t)+\omega^2\right)^2} + \left\{\beta^2\langle N\rangle^2A^2 - 4\langle N\rangle A^3\beta\right\}\int_{-\infty}^{\infty}\frac{d\omega}{\left(A^2(t)+\omega^2\right)^3}\right] \qquad \ldots (15)$$

Evaluating the integrals in (15), we have

$$\begin{aligned}\left\langle N_0^2(t)\right\rangle &= D_0\left[\frac{1}{A(t)} - \frac{\beta\langle N\rangle}{2A^2(t)} + \frac{3\beta^2\langle N\rangle^2}{8A^3(t)}\right]\\ &\simeq \frac{D_0}{\alpha}\left\langle 1+\frac{\beta\langle N\rangle}{\alpha}\right\rangle\\ &= \frac{D_0}{a}+\frac{D_0}{a^2}\beta\langle N\rangle\end{aligned} \qquad \ldots (16)$$

neglecting higher powers of $\beta<N>$. Then for small $\beta<N>$ under the condition (13), the result (16) for the quasisteady state agrees with the result (10) for large time *t*, as it should be.

3. VERHULST GROWTH UNDER RANDOM AND DETERMINISTIC DRIVING FORCES

A. Periodic Carrying Capacity : Linearization

Let us now consider the Verhulst equation (1) where the capacity *K* is now assumed to be time-dependent and this due to the variability of the environmental condition. Let us take *K*(*t*) to be of sinusoidal form

$$K(t) = K_0(1 + b\cos \omega t) \qquad \text{... (17)}$$

Then the equation (1) becomes

$$\frac{dN}{dt} = \alpha N\left[1 - \frac{N}{K_0\left(1 + b\cos\ \omega t\right)}\right] \qquad \text{... (18)}$$

The equation (18) can be scaled as follows

$$\text{We write } N' = \frac{N}{N_0},\ t' = \alpha t\cdot \qquad \text{... (19)}$$

Then the equation (18) reduces to the form

$$\frac{dN'}{d\,t'} = N'\left[\frac{N}{\left(1 + b\cos \omega' t'\right)}\right] \qquad \text{... (20)}$$

$$\text{where } \omega' = \frac{\omega}{\alpha} \qquad \text{... (21)}$$

The steady state value for N' as seen from (20) is given by

$$N' + 1 = b\cos \omega' t' \qquad \text{... (22)}$$

For $b \ll 1$ and for long time interval the steady value of *N'* is one. Then writing $N' = 1+y$ and expanding about the steady value, the linear equation that results from (20) is given by [5]

$$\frac{dy}{d\,t'} + y = b \cos\ \omega'\,t' \qquad \text{... (23)}$$

The equation (23) is the basic deterministic equation and is a linear equation in y with a driving time-dependent force $b \cos \omega' t'$.

B. Stochastic Differential Equattion : Fluctuation

The stochastic extension of (23) is obtained by addition of random perrturbation term $\varepsilon(t')$ to the right hand side of (23).

$$\frac{d\,t}{d\,t'} + y = b\,\omega'\,t' + \varepsilon(t') \qquad \text{... (24)}$$

Where random noise $\varepsilon(t')$ is assumed to be a gaussian white noise satisfying the conditions

$$\left\langle \varepsilon\left(t_1'\right)\right\rangle = 0,\ \left\langle \varepsilon\left(t_1'\right)\varepsilon\left(t_1'\right)\right\rangle = 2D\,\delta\left(t_1' - t_2'\right) \qquad \text{... (25)}$$

Where < > represents the average over the ensemble of the stochastic process. The equation (24) is an extension of Langevin equation driven by a periodic force $b \cos \omega' t'$ and a random force $\varepsilon(t')$. The stochastic behaviour of the system satisfying the equation (24) can be studied from the extended form of the famous 'fluctuation- dissipation theorem' of non-equilibrium statistical mechanics [4].

To find the correlation function for the system satisfying the stochastic equation (24) let us assume that the system has evolved from the state $x = 0$ at some very distance time past $t = -\infty$ under the action of the time-varying external force $b \cos \omega' t'$. Then following Lavenda [4] the auto - correlation function is given by

$$C\left(t', t_0'\right) = \left\langle y\left(t'\right) y\left(t_0'\right)\right\rangle$$

$$= D \exp\left\{-\left(t' - t_0'\right)\right\} = \int_{-\infty}^{t'} \exp\left\{-\left(t' - u\right)\right\} b \cos\ \dot{\omega} u\, du$$

$$\int_{-\infty}^{t_0'} \exp\left\{-\left(t_0'-u\right)\right\} b\cos\, \omega' u \, du \qquad \dots (26)$$

$$= D\exp\,\left(t_0'-t'\right)+b\,\frac{\left(\omega'\sin\,\omega' t' + \cos\,\omega' t'\right)}{1=\omega'}$$

$$b\,\frac{\left(\omega'\sin\,\omega' t_0' + \cos\,\omega' t_0'\right)}{1+\omega'} \qquad \dots (27)$$

From (27) the average fluctuation $\left\langle y^2\left(t'\right)\right\rangle$ is given by

$$\left\langle y^2\left(t'\right)\right\rangle = D + \frac{b^2}{\left(1+\omega'\right)^2}\left(\omega'\sin\omega' t' + \cos\,\omega' t'\right)^2 \qquad \dots (28)$$

which consists of two parts. The first part is due to the random force and the second part is due to the driving force, i.e., due to the environmental variability coming through the time-dependent carrying capacity. The average of $y\,(t')$ is given by

$$\left\langle y\left(t'\right)\right\rangle = \int_{-\infty}^{t'} \exp\left\{-\left(t'-u\right)\right\} b\cos\left(\omega' u\right) du$$

$$= \frac{b}{1+\omega'}\left(\omega'\sin\omega' t' + \cos\omega' t'\right) \qquad \dots (29)$$

which is the particular part of the solution of the deterministic equation (23), the transient part of the general solution vanishes for large value of the time t.

The mean square fluctuation of $y(t')$ is then given by

$$\sigma^2(t') = \langle y^2(t') \rangle - \langle y(t') \rangle^2 = D \qquad \ldots (30)$$

The expression shows that the driving force $b \cos \omega' t'$ for the linearized equation (23) has no effect on the dispersion of the growth variable. The fluctuation arises as a result of the random force only. This, infact, follows as a consequence of the Gaussian-Markovinian assumption.

4. Conclusion

The paper aims to study the stochastic behaviour of non-equilibrium fluctuation in logistic non-linear growth of a biological system by the technique of linearization. Two problems have been studied - one is the fluctuation for constant carrying capacity and the other for the periodic carrying capacity of the environment. The paper is concerned with an important biological application of famous 'fluctuaion dissipation theorem' of statistical mechanics. In the first case it is based on a modified form of Nyguist's theorem for quasi - steady state. The result obtained in this case is a modification of the earlier well established and experimentally verified result of population fluctuation for constant carrying capacity. In the second case of periodic carrying capacity the theory is based on the extended form of "fluctuation-dissipation-theorem" for driving system. The results obtained in this case are in good agreement with those of Nisbet and Gurney [5] who justified their results in the light of Jillson's experiment on the culture of Tribolium Castaneum.

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AN EXTENSION OF ANCIENT INDIAN CUBE-ROOT METHOD

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ABSTRACT

In this paper an attempt is made to extend Bhāskarācārya's method of obtaining cube-root of a perfect cubic number to 5th root of any integer. The method indeed may be applied to compute nth root of any positive integer.

1 : INTRODUCTION

Ancient Indian Mathematicians made an attempt to find out the cube-root of perfect numbers. For a detailed discussion refer to Līlāvatī of Bhāskara II (see also Bag [1], Datta and Singh [2], Jha [3], Mishra [7] and Naimpally *et al.* [8]. An attempt has been made by Lal [6] to give a chronological description of the methods for obtaining the real cube root of an integer given by the Hindu Mathematicians. Bhāskara II, in his Līlāvatī, gave a method to find out (real) cube-root of a positive integer. It seems that ancient Indian Pandits knew cube-root techniques much before Bhāskara (b.1114 A.D.). Bhāskara's method is slightly different from his predecessors.

In this paper we make an attempt to give a general method to compute real nth root of an integer. Apart from including several examples to illustrate the technique of computing 5th, 6th, 7th roots of a number,

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'quick computation' is also indicated in certain cases, particularly when the root to be computed is of two digits.

Definition : A number is said to be a perfect number of order n (or simply a perfect number in this paper) if it is expressible in the form of x^n, where n is a positive integer and x is any rational number.

First look at the following table. Calculations at various steps become a little easier if one remembers the message of this table.

TABLE -1

SHOWING END OF ROOTS OF PERFECT NUMBERS ACCORDING TO THEIR UNIT PLACES

Number	Unit places in different powers														
	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	2	4	8	6	2	4	8	6	2	4	8	6	2	4	8
3	3	9	7	1	3	9	7	1	3	9	7	1	3	9	7
4	4	6	4	6	4	6	4	6	4	6	4	6	4	6	4
5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
7	7	9	3	1	7	9	3	1	7	9	3	1	7	9	3
8	8	4	2	6	8	4	2	6	8	4	2	6	8	4	2
9	9	1	9	1	9	1	9	1	9	1	9	1	9	1	9

Note that :

1. 5th, 9th, 13th, 17th etc. powers of 1, 2, 3, 4, 5, 6, 7, 8 & 9 end up respectively in 1, 2, 3, 4, 5, 6, 7, 8 & 9.
2. 6th, 10th, 14th, 18th etc. powers of 1, 2, 3, 4, 5, 6, 7, 8 & 9 end up respectively in 1, 4, 9, 6, 5, 6, 9, 4 & 1.

3. 7th, 11th, 15th, 19th etc. powers of 1, 2, 3, 4, 5, 6, 7, 8 & 9 end up respectively in 1, 8, 7, 4, 5, 6, 3, 2, & 9.
4. 8th, 12th, 16th, 20th etc. powers of 1, 2, 3, 4, 5, 6, 7, 8 & 9 end up respectively in 1, 6, 1, 6, 5, 6, 1, 6 & 1.

5TH ROOT OF A POSITIVE INTEGER

Note that fifth power of number 1 through 9 are respectively 1, 32, 243, 1024, 3125, 7776, 16807, 32768 and 59049

Given a perfect positive integer of order 5, to find out its fifth positive real root.

Step I : Put a dash on the unit place and a dot on each of the next four digits. Repeat this process till the last digit of the number. Note that the number of dashes equals the number of digits (before decimal) in the proposed fifth root. For example, if the given number consists of 13 digits, then it is divided by the "dash-dot" method into 3 blocks and the first two blocks consist of 5 digits each and the last block consists of only 3 digits.

Step II : From the last block subtract the 5^{th} power of the greatest possible positive integer, say a.

Step III : Associate with this remainder, the first digit of the next block. Now divide this new number by $5a^4$. Let the quotient be b. Evidently $0 \le b \le 9$ (see Example 2.1 wherein 4 is associated with the remainder 111 to give a new number 1114, and b = 7). This new number may be called 2nd Pada (or Pada II), while the (last) block with which we started the actual computation may be called "first Pada" (or simply Pada I).

Step IV : Form Pada III by taking down this new remainder and the next digit. (For example, our third Pada in Example 2.1 is 5548). Subtract $10a^3b^2$ from Pada III, $10a^2b^3$ from Pada IV, $5ab^4$ from Pada V and b^5 from Pada VI. (If subtraction of $10a^3b^2$ is not possible, reduce b of Step III until subtraction becomes possible, and now the subtraction is done with the reduced b).

Step V : If more blocks are left out divide Pada VII by $5(10a+b)^4$ to get quotient c (where $c \leq 9$). Subtract $10\,(10a+b)^3c^2$ from Pada VIII, $10\,(10a+b)^2c^3$ from Pada IX, $5(10a+b)c^4$ from Pada X and c^5 from Pada XI respectively. (If subtraction of $10\,(10a+b)^3c^2$ is not possible, c is reduced and adjusted in the same way as b).

Step VI : If more blocks are still left out, divide the next Pada XII by $5\,(100a+10b+c)^4$ to get quotient d where d (≤ 9) is adjusted, if necessary, in the same way as b and c). Subtract $10\,(100a+10b+c)^3d^2$, $10(100a+10b+c)^2d^3$, $5\,(100a+10b+c)d^4$ and d^5 from the next Padas respectively.

The process may be extended in this fashion if more blocks are still left out.

Results : (i) For the number having two blocks root is 10a+b, (where b is at the unit place for perfect numbers only).

(ii) For a number having three blocks (respectively four blocks), root is 100a+10b+c (respectively 1000a+100b+ 10c+d) wherein c (respectively d) is at the unit place.

Now we illustrate the method by examples.

Example *2.1* Find the 5th real root of 14348907.

Computation may be done as follows wherein a and b assumed in Steps II-V are given in this and subsequent examples only for the sake of clarity.

$$\dot{1}\;\dot{4}\;\overset{'}{\dot{3}}\;\dot{4}\;\dot{8}\;\dot{9}\;\dot{0}\;\overset{'}{7}$$

$$\begin{array}{lll}
a^5 = 2^5 & & 2 = a \\
 & \;32 & \\
5a^4 = 5.2^4 \quad = & 80\overline{)\,1114\,(} & 7=b \\
5a^4b = 560 & \underline{\;560\;} & \\
10a^3\,b^2 = 10.2^3\,.7^2 & \;5548 & \\
\quad = 3920 & \underline{\;3920\;} & \\
 & \;16289 & \\
10a^2\,b^3 = 10.2^2.7^3 & & \\
\quad = 13720 & \underline{\;13720\;} & \\
 & \;25690 &
\end{array}$$

$5ab^4 = 5.2.7^4$	
$= 24010$	$\underline{24010}$
	16807
$b^5 = 7^5$	
$= 16807$	$\underline{16807}$
	×

Hence 5th root of 14348907 is 27.

It is important to note that, in view of the observations made on Table 1,7 seems to be at the unit place of the proposd 5th root. Since there are two blocks, the answer will comprise of only two digits. Consequently the Calculations beyond Pada II are not needed, if we know that the given number is a perfect number of 5th order.

Example 2.2 Find 5th root of 4747561509943.

We have the following unnarrated steps showing the details of calculations.

	$\dot{4}\dot{7}\;\overset{'}{4}\;\dot{7}5\dot{6}\dot{1}\;\dot{5}\;'\;\dot{0}\dot{9}9\dot{4}\;\dot{3}$
$a^5 = 3^5$	$3 = a$
	243
$5a^4 = 5.3^4 =$	$405\overline{)2317}(\ 4=b$
$10\,a^3b^2 = 10.3^2.4^2$	$\underline{1620}$
	6975
$= 4320$	$\underline{4320}$
$10\,a^2b^3 = 10.3^2.4^3$	26556
$= 5760$	$\underline{5760}$
$5ab^4 = 5.3.4^4$	207961
$= 3840$	$\underline{3840}$
	2041215
$b^5 = 4^5 = 1024$	1024
$5(10a + b)^4 = 5(34)^4 = 6681680$	$\overline{)20401910}(\ 3=c$
$10(10a+b)^3 c^2 = 10(34)^3 .3^2$	$\underline{20045040}$
	3568709
$= 3537360$	$\underline{3537360}$
$10(10s+b)^2 c^3 = 10(34)^2 .3^3$	313499
$= 312120$	$\underline{312120}$
$5(10a+b)\,c^4 = 5(34) . 3^4$	13794
$= 13770$	$\underline{13770}$
	$\overset{'}{2}43.$

$$\begin{array}{ll} c^5 = 3^5 = 243 & \underline{243} \\ & \times \end{array}$$

Hence 5th root of 4747561509943 is 343.

Example 2.3 : Find 5th root of 14693280768.

$$\overset{'}{1}\ \dot{4}\dot{6}\dot{9}\dot{3}\overset{'}{2}\ \dot{8}\dot{0}\dot{7}\dot{6}\overset{'}{8}$$

$$\begin{array}{lll} a^5 = 1^5 & & 1 = a \\ 5a^4 = 5.1^4 = & 5\overline{)46}^{\,1} & (08 = b \\ 10a^3\,b^2 = 10.1^3 \,.\, (08)^2 & \underline{40} & \\ & 693 & \\ \quad = 640 & \underline{640} & \\ 10a^2\,b^3 = 10\,(1)^2\,(08)^3 & 5328 & \\ \quad = 5120 & \underline{5120} & \\ 5ab^4 = 5.1.\,(08)^4 & 20807 & \\ \quad = 20480 & \underline{20480} & \\ b^5 = (08)^5 & 32768 & \\ \quad = 32768 & \underline{32768} & \\ & \times & \end{array}$$

Here 4 is not divisible by 5 and so we put 0 as quotiant and bring down the next digit 6. Note that 46 is our Pada II.

Hence 5th root of 14693280768 is 108.

3 : 5th ROOT METHOD FOR NON-PERFECT NUMBERS

Step I : Put dashes and dots as mentioned earlier to subdivide the number into blocks of five.

Step II : Put decimal and zeros after the unit place.

Example 3.1 : Find 5th root of 2.

We do it as before.

$a^5 = 1^5$ 2.00000
1
$5a^4 = 5.1^4 =$ 5) 10 (1=*b* 1 = a
5
$10a^3 b^2 = 10. (1)^3. (1)^2$ 50
$= 10$ 10
400
$10a^2 b^3 = 10.1^2.1^3 = 10$ 10
3900
$5ab^2 = 5.1.1^4 = 5$ 5
38950
$b^5 = 1^5 = 1$ 1
$5\,(10a + b)^4 = 5.\,(11)^4 = 73205$)389490 (5=c
366025
23465 (Remainder)

Hence 5th root of 2 (till two places of decimal) is 1.15.

Example 3.2 : Find 5th root of 68949

We follow again the previous method.

68949 .000000
$a^5 = 9^5$ 59049 9 = a
$5a^4 = 5.9^4 =$ 32805)99000 (2=*b*
65610
333900
$10a^3 b^2 = 10.9^3.2^2 = 29160$ 29160
3047400
$10a^2b^3 = 10.9^2.2^3 = 6480$ 6480
30409200
$5ab^4 = 5.9.2^4 = 720$ 720
304084800
$b^5 = 2^5 = 32$ 32

$$\begin{array}{lrl}
5\,(10a+b)^4 = 5(92)^4 = & 177065680 & \overline{)\,3040847680\,}(\,9 = c \\
 & & \underline{\;1593591120\;} \\
 & & \;\;14472565600 \text{ (Remainder)}
\end{array}$$

We were looking for 9.29 (correct to two places of decimal).

Example 3.3 : Find 5th root of 34867. 84401.
(Given that this is a perfect number of order 5.)

$$\begin{array}{lrll}
 & & \dot{3}\dot{4}\dot{8}\dot{6}\overset{'}{7}.\,8\,4\,4\,0\,1 & \\
a^5 = 8^5 & & 32768 & 8 = a \\
5a^4 = 5.8^4 = & 20480 & \overline{)\,20998\,}(\,1 = b & \\
 & & \underline{20480} & \\
10a^3\,b^2 = 10.\,8^3\,.\,1^2 & & 5184 & \\
\quad = 5120 & & \underline{5120} & \\
10a^2\,b^3 = 10.\,8^2\,.\,1^3 & & 644 & \\
\quad = 640 & & \underline{640} & \\
5ab^4 = 5.\,8\,.\,1^4 & & 40 & \\
\quad = 40 & & \underline{40} & \\
 & & 1 & \\
b^5 = 1^5 & & \underline{1} & \\
 & & \times &
\end{array}$$

Hence 5th root of 34867. 84401 is 8.1

4 : 6TH ROOT

Example 4.1 : Find 6th root of 191102976.

$$\begin{array}{lrl}
 & & \dot{1}\dot{9}\;\overset{'}{1}\;\dot{1}\dot{0}\dot{2}\dot{9}\dot{7}\,\dot{6}\;' \\
a^6 = 2^6 & & 64 \\
6a^5 = 6.2^5 = & 192 & \overline{)\,1271\,}(\,4 = b \\
 & & \underline{768} \\
15a^4\,b^2 = 15.2^4.4^2 & & 5030 \\
\quad = 3840 & & \underline{3840} \\
 & & 11902 \\
20a^3b^3 = 20.2^3.4^3 = 10240 & & \underline{10240} \\
 & & 16629 \\
15a^2\,b^4 = 15.2^2.4^4 = 15360 & & \underline{15360} \\
 & & 12697
\end{array}$$

$6a.b^5 = 6.2.4^5 = 12288$	$\underline{12288}$	
	4096	
$b^6 = 4^6 = 4096$	$\underline{4096}$	
	×	

Hence 6th root of 191102976 is 24.

Example 4.2 : Find 6th root of 17685.

	$\dot{1}\dot{7}\dot{6}\dot{8}\dot{5}.000000$	
$a^6 = 5^6$	15625	5 = a
$6a^5 = 6.5^5 = 18750$	$18750\overline{)\,20600}\,(1=b$	
	$\underline{18750}$	
	18500	
$15a^4b^2 = 1^5.5^4.1^2 = 9375$	$\underline{9375}$	
	91250	
$20\,a^3b^3 = 20.5^2.1^3 = 2500$	$\underline{2500}$	
	887500	
$15a^2b^4 = 15.5^2.1^4 = 375$	$\underline{375}$	
	8871250	
$6\,a.b^5 = 6.5.1^5 = 30$	$\underline{30}$	
	88712200	
$b^6 = 1^6 = 1$	1	
$\dot{6}\,(10a + b)^5 = 6\,(51)^5 =$	$1274551506\overline{)\,8871219900}\,(\;06=c$	
	$\underline{7647309036}$	(Remainder)
	1223910864	

Hence 6th root of 17685 is 5.106

5 : 7TH ROOT

Example 5.1 : Find 7th root of 170859375

	$\dot{1}\;\dot{7}\;\dot{0}\dot{8}\dot{5}\dot{9}\dot{3}\dot{7}\dot{5}$	
$a^7 = 1^7$	1	1 = a
$7a^6 = 7.1^6 = 7$	$7\overline{)\,160}\,(\,5=b$	
	$\underline{35}$	
	1258	
$21\,a^5.b^2 = 21.1^5.5^2 = 525$	$\underline{525}$	
	7335	
$35a^4b^3 = 3^5.\,1^4\,.\,5^3 = 4375$	$\underline{4375}$	

$$\begin{array}{lr}
 & 29609 \\
35a^3b^4 = 35 \,.\, 1^3 \,.\, 5^4 = 21875 & \underline{21875} \\
 & 77343 \\
21\, a^2b^5 = 21 \,.\, 1^2 \,.\, 5^5 = 65625 & \underline{65625} \\
 & 117187 \\
7ab^6 = 7.\, 1 \,.\, 5^6 = 109375 & \underline{109375} \\
 & 78125 \\
b^7 = 5^7 = 78125 & \underline{78125} \\
 & \times
\end{array}$$

Hence 7th root of 170859375 is 15.

n^{th} ROOT

Methods discussed above may be extended to obtaining n^{th} root (wherein n is any positive integer) of a number, say N. Subdivide the digits of N into blocks of n digits by putting a dash on the digit at the first place (i.e. at the unit place) and a dot on each of the next (n-1) digits. Continue this process. The last block, which is our Pada I, might not consist of n digits. Now subtract from the Pada I the n^{th} power of the greatest possible digit. Now form Pada II by associating the first digit of the last but one block with the remainder. Now divide Pada II by $na^{n-1} = {}^nC_1a^{n-1}$* . Assume that the quotient is b. form Pada III by associating the next digit of the last but one block with the remainder obtained on dividing by na^{n-1}. Now subtract from Pada III ${}^nC_2a^{n-1}b$. Continue this process of subtraction from the subsequent Padas up to ${}^nC_nb^n$. If the digits of N have not been exhausted, continue the whole process starting with $n(10a+b)^{n-1}$ instead of a . See Example 2.2.

Indeed the method is some what an inversion of the Binomial Theorem:

$$(a+b)^3 = a^3 + 3a^2b+3ab^2+b^3$$

$$(a+b)^4 = a^4+ 4a^3b+6a^2b^2+4ab^3+b^4$$

$$(a+b)^5 = a^5+5a^4b+10a^3b^2+10a^2b^3 + 5ab^4+b^5$$

---

* ${}^nC_r$ stands for n! / r ! (n– r) !

……………………………………………………

$$(a+b)^n=a^n+{}^nC_1a^{n-1}b+{}^nC\,a^{n-2}b^2+\dots+{}^nC_n+b^n$$

In order to make the 'inversion process' more tangible, we obtain below 6th root of an algebraic expression. For the sake of simplicity we find out 6th root of ($a^6+6a^5b+15a^4b^2+20a^3b^3+15a^2b^4+6ab^5+b^6$). The following is the calculation chart:

$$a^6+6a^5b+15a^4b^2+20a^3b^3+15a^2b^4+6ab^5+b^6$$

$$=a$$

$$\begin{array}{ll}
 & \underline{a^6} \\
6a^5 &)\,6a^5b\ (\,b \\
 & \underline{6a^5b} \\
 & 15a^4b^2 \\
15a^4b^2 & \underline{15a^4b^2} \\
 & 20a^3b^3 \\
20a^3b^3 & \underline{20a^3b^3} \\
 & 15a^3b^4 \\
15a^2b^4 & \underline{15a^2b^4} \\
 & 6ab^5 \\
6ab^5 & \underline{6ab^5} \\
 & b^6 \\
b^6 & \underline{b^6} \\
 & \times
\end{array}$$

$(a^6+6a^5b+15a^4b^2+20a^3b^3+15a^2b^4+6ab^5+b^6)1/6 = a+b$

Note that in this algebraic expression 6th root will be a+b and not 10a+b since a,b are not restricted to digits only.

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PRESERVATION OF GENERALIZED CONTINUITY

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ABSTRACT

The main purpose of this paper is to study the effects of uniform convergence and its various generalizations on functions which are nearly-continuous or quasi-continuous. It is known that uniform convergence preserves near continuity as well as quasi-continuity. In this paper a general result is proved to the effect that uniform convergence preserves several weaker forms of continuity including near and quasi continuity. Investigations are also made concerning Dini convergence, Leader convergence, Simple uniform convergence etc. in relation to functions which are nearly or quasi continuous. Necessary tools in the form of pretopology and preproximity are also developed.

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KEY WORDS AND PHRASES

Uniform convergence, Arzelà convergence, Dini convergence, Leader convergence, Quasi-uniform convergence, Simple uniform convergence, Simple Leader convergence, Semi-R_0, Semicompact, Pretopology, Preproximity, Near c** ontinuity, Quasi-continuity, Correct uniformity.

1. INTRODUCTION

Among the various generalizations of continuity, besides upper and lowr semicontinuity, two stand out viz.. *near continuity* (which Blumberg [3] called *densely approached*) and *quasi-continuity* (Kempisty [9]). Near continuity occurs naturally in problems involving open mapping and closed graph theorems of Functional Analysis [13]. On the other hand quasi-continuity is at the basis of results involving joint versus separate continuity. Hence it is no wonder that these two generalizations of continuity have been rediscovered several times under differenent names! In this paper we propose to study the effects of various convergences, which are generalizations of uniform convergence, on these two weak forms of continuity. For a study of continuous functions the reader is referred to (Di Concilio-Naimpally [4]).

Suppose (X, τ) and (Y, σ) are nonempty topological spaces. When we wish to discuss uniform convergence or generalisations we naturally assume that σ is Tychonoff and has a compatible (generalized) uniformity as well as a compatible EF proximity δ_2.

(1.1) *Definition* : A function $f : (X, \tau) \to (Y, \sigma)$ is **quasi continuous** at $x_0 \in X$ iff for each $U \in \tau$, $V \in \sigma$, $x_0 \in U$, $f(x_0) \in V$ there is a nonempty set $W \in \tau$ such that $W \subset U$ and $f(W) \subset V$. Clearly f is quasi continuous iff for each $V \in \sigma$, $f^{-1}(V) \subset f^{-1}(V)^{0-}$. where 0 and - are interior and closure operators respectively (Kempisty [19]).

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(1.2) *Definition* : A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is **nearly continuous** at $x_0 \in X$ iff for each $V \in \sigma$ containing $f(x_0)$, there is a nbhd. U of x_0 such that $U \subset f^{-1}(V)^-$. Obviously f is nearly continuous iff for each $V \in \sigma$, $f^{-1}(V) f^{-1}(V)^{-0}$.

Although there is a vast literature on quasi continuity and near continuity, it is not widely known that they together characterize continuity when (Y, σ) is regular.

(1.3) *Theorem.* (Garg-Naimpally [6]) If $f : (X, \tau) \rightarrow (Y, \sigma)$ is quasi continuous and nearly continuous then f is θ-continuous. Hence if (Y, σ) is regular, then f is continuous if and only if f is quasi-continuous and nearly continuous

Now we show how quasi continuous and nearly continuous functions can be considered as continuous when the topology τ on the domain X is replaced by larger families called *pretopologies*. From the definitions (1.1) and (1.2) it is clear that the following families are associated with quasi continuous and nearly continuous functions respectively :

(1.4) $$\tau_q = \{A \subset X : A \subset A^{0-}\}$$

(1.5) $$\tau_n = \{A \subset X : A \subset A^{-0}\}$$

It is easy to check that τ_q and τ_n both contain τ and are closed under *arbitrary unions* but not necessarily under *finite intersections*. This leads us to the following definition :

(1.6) *Definition* : A family of subsets ρ of X is a **pretopology** iff ρ contains ϕ, X and is closed under arbitrary unions. (X, ρ) is called a **pretopological space.**

(1.7) *Remarks*. It is now obvious that

(a) $f : (X, \tau) \to (Y, \sigma)$ is quasi continuous iff $f : (X, \tau_q) \to (Y, \sigma)$ is continuous.

(b) $f : (X, \tau) \to (Y, \sigma)$ is nearly continuous iff $f : (X, \tau_n) \to (Y, \sigma)$ is continuous.

We now show that uniform convergence preserves continuity even when the domain space is pretopological. This result includes, as special cases, the results of Bledsoe [2] concerning neighborly i.e. quasi-continuous functions and Husain [8] concerning almost i.e. nearly continuous functions.

(1.8) *Theorem.* (Garg-Naimpally) Suppose (X, ρ) is a pretopological space and (Y, ν) an AN uniform space. If a net of continuous functions $\{f_n : n \in D\}$ converges uniformly to a function f, then f in continuous.

Proof. (We may suppose that all entourages are symmetric in this and all subsequent proofs.) For $V \in \nu$ there is a $V_1 \in \nu$ such that $V_1^3 \subset V$. Since $f_n \to f$, there is an $m \in D$ such that for all $n > m$ and for each $x \in X$, $f_n(x) \in V_1[f(x)]$. Since each f_n is continuous at $x_0 \in X$, there is $U \in \tau$ such that $x_0 \in U$ and $f_n(U) \subset V_1[f_n(x_0)]$ where we may suppose that $n > m$ then $f(U) \subset V_1^3[f(x_0)] \subset V\{fx_0)]$. Hence f is continous at x_0.

(1.9) *Corollary.* Uniform convergence preserves quasi-continuity (Bledsoe [2]) and near continuity (Husain [8]).

2. PRETOPOLOGY AND PREPROXIMITY.

If (X, ρ) is a pretopological space, then a set C is *closed* iff $X - C \in \rho$. We now define a *preclosure operator* $\mathrm{pcl} : P(X) \to P(X)$:

(2.1) *Definition*: $\mathrm{pcl}(A) = \cap \{C : A \subset C, X - C \in \rho\}$.

The following result is obvious :

(2.2) *Theorem* : The preclousre opertaor pcl on a pretopological space (X, ρ) satisfies:

(a) $\mathrm{pcl}(\phi) = \phi$,

(b) $A \subset \mathrm{pcl}(A)$ for each $A \subset X$,

(c) $\mathrm{pcl}(\mathrm{pcl}(A)) = \mathrm{pcl}(A)$,

(d) $A \subset B$ implies $\mathrm{pcl}(A) \subset \mathrm{pcl}(B)$.

(2.3) *Remarks.* Of course, pcl does not satisfy the union axiom but it satisfies the weaker axiom (d) which is equivalent to (d') $\mathrm{pcl}(A) \cup \mathrm{pcl}(B) \subset \mathrm{pcl}(A \cup B)$.

The next result is easy to prove.

(2.4) *Lemma.* Let (X, ρ) be a pretopological space. Then, for each $A \subset X$, $\mathrm{pcl}(A) = \{x \in X : \text{for each } U \in \rho, x \in U \Rightarrow U \cap A \neq \phi\}$

Furthermore, every preclosure oeprator pcl satisfying 2.2 (a) - (d), induces a pretopology ρ' where

$\rho' = \{U : \mathrm{pcl}(X - U) = X - U\}$.

Notation. $\underline{A} = \mathrm{pcl}(A)$.

We now define a preproximity.

(2.5) *Definition :* A binary relation $\not\delta$ on $P(X)$ is called a **basic preproximity** iff it satisfies.

(a) $A \not\delta B$ implies $B \not\delta A$,

(b) $A \not\delta B$ implies $A \neq \phi$, $B \neq \phi$,

(c) $A \cap B \neq \phi$ implies $A \not\delta B$,

(d) $A \not\delta B$ and $B \subset C$ implies $A \not\delta C$

A basic preproximity $\not\delta$ is called an **S-preproximity** if it satisfies :

(S) $a \delta B$ and $b \not\delta C$ for each $b \in B$, then $a \not\delta C$.

A basic preproximity δ is a **LO-Preproximity** if it satisfies:

(LO) $A \delta B$ and $b \delta C$ for each $b \in B$ implies $A \delta C$.

A basic preproximity is an **R-preproximity** if it satisfies :

(R) $a \delta B$ implies there is an $E \subset X$ such that $a \delta E$ and $(X - E) \delta B$.

(2.6) *Theorem* : The basic preproximity δ on X induces an operator δ on P(X) to P(X):

$$A^{\delta} = \{ x \in X : x \delta A \} \tag{2.7}$$

The above defines a preclosure operator if an only if δ is an S-preproximity. The pretopology induced by a preproximity δ is denoted by $\rho(\delta)$. We say that δ and $\rho(\delta)$ are **compatible.**

(2.8) *Definition* : A pretopological space (X, ρ) is **(symmetric)** R_0 iff $x \in y$ implies $y \in \underline{x}$.

The following results are proven in a similar manner to the corresponding results on proximities and topological spaces [11].

(2.9) *Theorem*. Every separated LO preproximity space (X, δ) induces a T_1 pretopological space $(X, \rho(\delta))$. Conversely, every T_1 pretopological space (X, ρ) has the finest compatible LO- preproximity δ_0 as well as the coarsest compatible LO- preproximity δ_{CL} where

$$A \delta_0 B \text{ iff } \underline{A} \cap \underline{B} \neq \phi , \tag{2.10}$$

$$A \delta_{CL} B \text{ iff } A \delta_0 B \text{ or both } \underline{A}, \underline{B} \text{ are infinite.} \tag{2.11}$$

(2.12) *Theorem*. If δ is a compatible LO-preproximity on a pretopological space (X, ρ), then

$$A \delta B \text{ iff } \underline{A} \delta \underline{B}$$

(2.13) *Definition.* A pretopological space (X, ρ) is **regular** iff for each x not in a preclosed set C, there are $U, V \in \rho$, $x \in U$, $C \subset V$ and $U \cap V = \phi$.

(2.14) *Theorem.* A pretopological space (X, ρ) is regular if and only if it has a compatible LO-R preproximity. The coarsest compatible R-S-preproximity δ_{CS} is given by

(2.15) $A \, \delta_{CS} \, B$ iff $(A \cap \underline{B}) \cup (\underline{A} \cap B) \neq \phi$. or A, B are both infinite.

The finest compatible S preproximity δ_S is given by

(2.16) $A \, \delta_S \, B$ iff $(\underline{A} \cap B) \cup (A \cap \underline{B}) \neq \phi$.

3. **QUASI-CONTINUITY.** In this section we pursue the limits of nets of quasi-continuous functions under various convergences. If (X, τ) is a topological space, then members of τ_q (1.4) are called **semi-open** and the resulting preclosure operator is called scl (**semi-closure operator**). In this section $\underline{A}$ denotes scl (A) and (X, τ) is called **semi - R_0** iff (X, τ_q) is R_0 (2.8). Such a space has compatible LO-preproximities defined by (2.10), (2.11) and if (X, τ_q) is regular then it has compatible S-preproximities defined by (2.15), (2.16).

In this section (X, τ) is a semi-R_0 topological space and δ_1 is any preproximity compatible with (X, τ_q). (Y, σ) dentoes an R_0 topological space with a compatible proximity δ_2. $(f_n : n \in D)$ denotes a net of functions on X to Y.

(3.1) *Definition.* A function $f : (X, \delta_1) \rightarrow (Y, \delta_2)$ is **p-quasi-continuous** iff whenever $A \, \delta_1 \, B$ in X, $f(A) \, \delta_2 \, f(B)$ in Y.

(3.2) *Remarks.* It is easy to see that a p-quasi-continous function is quasi-continuous. The following example shows that the converse is not true. Suppose $X = Y = [-1, 1]$ with the usual topology τ. Let δ_2 be the usual metric proximity on Y and δ_1 the coarsest preproximity δ_{CL} defined by (2.11).

$$f(x) = \begin{cases} 0 & at\ x = 0 \\ \sin 1/x & at\ x \neq 0 \end{cases}$$

Then f is quasi-continuous but not p-quasi-continuous. If $A = \{(n\pi)^{-1} : n \in N\}$, $B = \{[(2n+1)\pi/2]^{-1} : n \in N\}$, then $A\ \delta_1\ B$ but $f(A)\ \delta_2\ f(B)$.

For definitions of various convergences such as R.C., L.C., D.C., S.U.C. Q.U.C., S.L.C. and A.C. please see [4]. Here we merely give examples and state results which are not in [4].

(3.3) *Theorem* : If $f_n \xrightarrow{R.C.} j$ and each f_n is (p-) quasi-continuous, then f is (p-) quasi-continuous.

(3.4) *Example*. Let X = Y = R with the usual topology. For each $x \in X$ and $n \in N$ we set:

$$f(x) = \begin{cases} 1 & x \geq 0 \\ -1 & x < 0 \end{cases}$$

$$f_n(x) = f(x) + n.$$

Then f_n and f are quasi-continuous functions, $f_n \xrightarrow{R.C.} f$ but $f_n \not\xrightarrow{P.C.} f$. Since P.C. does not necessarily preserve quasi-continuity (Levine [10]), it follows that P.C. and R.C. are independent.

(3.5) *Corollary*, X.C. preserves (p-) quasi-continuity.

(3.6) *Theorem* : If δ_2 is EF and $f_n \xrightarrow{L.C.} f$ then $f_n \xrightarrow{X.C.} f$; hence LC. preserves (p-) quasi-continuity.

(3.7) *Example*. Suppose X = Y = R with the usual topology. For each $x \in X$ and $n \in N$ we set :

$$f(x) = \begin{cases} x^2 - 1 & x < 0 \\ x^2 & x \geq 0 \end{cases}$$

$$f_n(x) = \left(f(x) + \frac{1}{n}\right)^2$$

Clearly $f_n \xrightarrow{P.C.} f$ but $f_n \not\xrightarrow{L.C.} f$. for $A = N-\{1\}$, $B = \{(n+1/n)^2 : n \in N]$, $f(A) \not\delta_2 B$ but $f_n(A) \cap B \neq \phi$ for each $n \in N$ and so $f_n(A) \delta_2 B$. If δ_2 is the coarsest compatible S-proximity on R, then $f_n \xrightarrow{L.C.} f$. and $f_n \xrightarrow{X.C.} f$. Furthermore, if $\not\delta_2$ is the usual EF proximity,then $f_n \xrightarrow{X.C.} f$.

(3.8) *Example* : Suppose X= Y= [-1, 1] with the usual topoloyg δ_1 the finest compatible preproximity (2.10) on (X, τ_q) and $\not\delta_2$ the unique compatible EF proximity on Y. For each $x \in X$, $n \in N$ we define :

$$f(x) \equiv 0$$

$$f_n(x) = \begin{cases} 0 & x = 0 \\ \dfrac{n \sin(1/x)}{1 + n^2 \sin^2(1/x)} & otherwise \end{cases}$$

Here $f_n \xrightarrow{P.C.} f$. and $f_n \xrightarrow{R.C.} f$ and so $f_n \xrightarrow{X.C.} f$. But there exists a sequence $(x_n) \in X$ such that $\sin(1/x_n) = {}^1/_n$. Set $A=\{x_n\}$, $B = \{1/2\}$; then $f(A) \not\delta_2 B$ but $f_n(A) \delta_2 B$ for each $n \in N$. Thus $f_n \not\xrightarrow{L.C.} f$. .

(3.9) *Example* : The example (2.9) (c) of Di Concilio-Naimpally [4] shows that the sequence

$$f_n(x) = \begin{cases} x + 2/n & x \leq 0 \\ x + 1/n & x > 0 \end{cases}$$

is a squence of quasi-continuous functions which L.C. converges to the continuous function $f(x) = x$ in the usual proximity. But if $\not\delta_r$ is the R-proximity defined by

(3.10) $\quad A \delta_r B$ iff $(A \cap B^-) \cup (A^- \cap B) \neq \phi$.

then $f_n \not\xrightarrow{X.C.} f$. and $f_n \not\xrightarrow{L.C.} f$.

(3.11) *Theorem* : If (X, τ) is semi-compact (i. e) every semi-open cover of X admits a finite subcover [14] and $f_n \xrightarrow{S.L.C.} f$ which is quasi-continuous, then $f_n \xrightarrow{Q.U.C.} f$.

(3.12) *Theorem* : Suppose X is semi-compact and δ_2 is EF. If $f_n \xrightarrow{P.C.} f$ and each f_n and f are quasi-continuous, then $f_n \xrightarrow{A.C.} f$.

Proof. Suppose f is quasi-continuous, $p \in X$, $B \subset X$, and $f(p)\, \delta_2\, f(B)^-$. Since f is quasi-continuous, $f(\underline{B}) \subset f(B)^-$ and since X is semi-compact we may suppose that B is semi-compact. Since δ_2 is EF, there is an open set $V \subset Y$ such the $f(B) \subset V$ and $f(p)\, \not\delta_2\, V$. Suppose $m \in D$. Since $f_n \xrightarrow{P.C.} f$., there exists an $m' > m$ such that $f_n(p)\, \not\delta_2\, V$ for each $n > m'$. Also we have $B \subset f_{n_i}^{-1}(V)$ for each $n > m'$. Since B is semi-compact and each $f_n^{-1}(V)$ is semi-open, $B \subset \bigcup_{i \in I_m} f_{n i}^{-1}(V)$. Consequently,

$$B = \bigcup_{i \in I_m} B_i \text{ where } B_i = B \cap f_{n_i}^{-1}(V).$$

Hence $f_{n i}(p)\, \not\delta_2\, f_{n i}(B_i)$ i.e. $f_n \xrightarrow{A.C.} f$.

(3.13) *Theorem* : If X is semi-compact and extremally disconnected, δ_2 if EF, $f_n \xrightarrow{A.C.} f$, each f_n is quasi-continuous implies f is quasi-continuous.

Proof. In this case the semi-closure operator (Njastad [12]) is a Kuratowski closure operator and we follow the proof of Di Concilio-Namipally [4].

4. NEAR CONTINUITY : In this section we study the convergence of nearly continuous functions. Since several of the results of the last section go through easily for nearly continuous functions as well, we mostly present counter examples in this section.

If (X, τ) is a topological space, then τ_n as defined by (1.5) consists of nearly open sets [13]. Complements of the members of τ_n are called **nearly closed** and they naturally induce a preclosure operator ncl. We write $\underline{A}$ = ncl (A). Suppose (X, τ) is such that ncl {x} = {x} for each x $\in$ X. Then (X, τ_n) has a compatible LO preproximity δ_0 defined by

$$A\ \delta_0\ B \text{ iff } \operatorname{ncl}(A) \cap \operatorname{ncl}(B) \neq \phi$$

If (Y, σ), is a topological space, then f : (X, τ) $\rightarrow$ (Y, σ) is nearly continuous iff

$$f(\underline{A}) \subset f(A)^- \text{ for each } A \subset X.$$

Definition of a p-nearly continuous function is similar to that of a p-quasi continuous function.

4.1*Example*. We now present an example of a nearly continuous function which is not p-nearly continuous. X = Y = R with the usual topology.

$$f(x) = \begin{cases} 0 & x \in Q \\ 1 & otherwise \end{cases}$$

If δ_{CL} denotes the coarsest compatible LO preproximity on (X, τ_n), then

A,δ_{CL} B but f(A) δ_2 f(B)

Where A = {n : n $\in$ N}, B = {n + $\sqrt{2}$: n $\in$ N} and δ_2 is the usual metric proximity on Y.

(4.2) *Definition.* A topological space (X, τ) is strongly compact iff every nearly open cover of X has a finite subcover [1].

Results analogous to (3.3) and (3.6) are ture for nearly continuous functions. Analogue of Theorem (3.11) also holds if we replace semi-compact by strongly compact. Similarly semi-compact is to be replaced by strongly compact in (3.12).

(4.3) *Lemma.* (Ganster [5]) Let X = F ∪ G be the Hewitt representation of X, where F is closed and resolvable, G is open and hereditarily irresolvable with $F \cap G = \phi$. The following are equivalent,

(a) (X, τ_n) is a topology

(b) G is open and each point of F^o is nearly open.

Analogue of Theorem (3.13) is true if we replace semi-compact by strongly compact and extremally disconnected by (a) or (b) of Lemma (4.3).

We now present counterexamples :

(4.4) *Example*. X = Y = R with the usual topology.

$$f(x) = \begin{cases} 0 & x \in Q \\ 1 & \text{otherwise} \end{cases}$$

$f_n(x) = f(x) + n.$

Then $f_n \xrightarrow{R.C.} f$ but $f_n \not\xrightarrow{P.C.} f$. Since Husain [8] has shown that P.C. does not preserve near continuity, we have P.C. and R.C. independent.

(4.5) *Example* . X = Y= [-1, 1] with the usual topology.

$f(x) = 0$

$$f_n(x) = \begin{cases} 1/n & x \in X-Q \\ nx/(1+n^2x^2) & \text{otherwise.} \end{cases}$$

Then $f_n \xrightarrow{P.C.} f$., $f_n \xrightarrow{R.C.} f$ but $f_n \not\xrightarrow{L.C.} f$, since if $A = \{1/n : n \in N\}$; $B = \{1/2\}$, $f(A) \not\delta_2 B$ but $f_n(A)\ \delta_2\ B$.

(4.6) *Example*. X =Y= R with the usual topology.

$$f(x) = \begin{cases} x^3 & x \in Q \\ -x^3 & otherwise \end{cases}$$

$$f_n(x) = \begin{cases} \left(x + \frac{1}{n}\right)^3 & x \in Q \\ -\left(x + \frac{1}{n}\right)^3 & otherwise. \end{cases}$$

Set $A = \{n \in N - \{1\}\}$, $B = \{n + 1/n)^3 : n \in N - \{1\}\}$. Then $f(A) \not\delta_2$ B but $f_n(A) \cap B \neq \phi$ for each $n \in N$. Consequently, $f_n \xrightarrow{L.C.}\!\!\!\!\!\not\;\;\; f$. But if δ'_2 is the R-proximity defined by (3.10), then $f_n \xrightarrow{L.C.} f$. However, $f_n \xrightarrow{X.C.} f$ if δ_2 is EF or the above.

(4.7) *Example* . X = Y = R with the usual topology.

$$f(x) = \begin{cases} x & x \in Q \\ 0 & otherwise \end{cases}$$

$$f_n(x) = \begin{cases} x + 1/n & x \in Q - \cup \{O\} \\ x + 2/n & x \in Q + \\ 0 & otherwise. \end{cases}$$

If δ_2 is the usual metric proximity, then $f_n \xrightarrow{L.C.} f$. But if δ'_2 is (3.10), then $f_n \not\to f$ in X.C. or L.C.

CONCLUSION : We have already noted that the family of nearly open sets or the family of semi-open sets does not form a topology for R. We recall the R with the usual topology is anti-semicompact i.e. every semicompact subset is finite [14]. Furthermore, no closed interval of R can be strongly compact. Hence we cannot have Arzela-type convergence for quasi-continuous or nearly continuous functions on a closed bounded interval of R.

The techniques developed in this paper can also be used to study the convergence of other generalized forms of continuity viz. *weak continuity, complete continuity, α-continuity, supercontinuity* (for definitions see [7]). In fact it is not difficult to set up, for each of these forms of generalized continuity, an appropriate pretopology and an associated LO-preproximity on the domain. Then we can show that these generalized forms of continuity are preserved under U.C., D.C., L.C. and X. C. It is also not difficult to see that U.C. preserves feeble continuity [7].

However, to attack the problem of preservation of *strong continuity, irresoluteness and δ-Continuity* [7], we are forced to set a pre-topology and a pre-proximity on both the domain and the range. In this manner, we can show that irresoluteness, strong continuity and δ-continuity are preserved only under X.C. and D.C.

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A NOTE ON THE STRESS DISTRIBUTION IN A ROTATING CIRCULAR DISK OF NONHOMOGENEOUS MATERIAL UNDER TRANSIENT SHEARING FORCE

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ABSTRACT

In this note, distribution of stress in a nonhomogeneous annular circular plate rotating with time dependent angular velocity has been found. Numerical results have been presented to study the effect of nonhomogeneity on stress components

INTRODUCTION

The problem of stress distribution in a steadily rotating circular disk has been gaining considerable interest among the research workers for its applications in engineering fields. In literature we find many investigators concentrating their attention on the problem under different conditions. The works of Singh and Puri [1], Samanta [2], Chakravorty and Chaudhuri [3] may be referred in this connection. In the present note my aim is to study the problem assuming the material of the disk as nonhomogeneous in the sense that both the Young's modulus E and the density ρ of the material are functions of position. It is also assumed that the disk is rotating with exponentially decaying time-dependent angular velocity. The problem has been completely solved and the expressions for the stresses have been given in terms of infinite series. Finally, to study the effect of nonhomogeneity on the stresses numerical computations have been done and the results have been graphically exhibitted.

FORMULATION OF THE PROBLEM

We assume that the stresses and displacements do not vary across the thickness of the disk and the lateral surface is free from any stress. Taking the centre of the circular plate as pole and using polar coordinates (r, θ) we find that for reasons of symmetry, the radial displacement U_r and the tangential displacement U_θ are independent of θ.

The equations of motion are

$$\frac{\partial \sigma_r}{\partial r}+\frac{\sigma_r-\sigma_\theta}{r}+\rho\, \Omega^2 r=\rho\, \frac{\partial^2 U_r}{\partial t^2} \tag{1}$$

$$\frac{\partial \tau_{r\theta}}{\partial r}+\frac{2\tau_{r\theta}}{r}-\rho\, \frac{d\Omega}{dt} r=\rho\, \frac{\partial^2 U_r}{\partial t^2}$$

where σ_r, σ_θ and $\tau_{r\theta}$ are the radial, tangential and the shearing stress components respectively given by

$$\sigma_r=\frac{E}{1-\upsilon^2}\left[\frac{\partial U_r}{\partial r}+\nu\, \frac{U_r}{r}\right]$$

$$\sigma_\theta=\frac{E}{1-\upsilon^2}\left[\upsilon\, \frac{\partial U_r}{\partial r}+\frac{U_r}{r}\right] \tag{2}$$

$$\tau_{r\theta}=\frac{E}{2\,(1+\upsilon)}\left[\frac{\partial U_\theta}{\partial r}-\frac{U_\theta}{r}\right]$$

$E = E(r)$ is the Young's modulus, $\rho = \rho(r)$ is the variable density, ν is the Poisson's ratio of the material and $\Omega = \Omega(t)$ is the angular velocity of the disk. We assume that Ω varies exponentially with time such that

$$\Omega=\Omega_0 e^{-p\tau}$$

where Ω_0 and p $(>O)$ are constants and $\tau \;\; t/t_0$

The boundary conditions are

$$\tau_{r\theta} = -M e^{-pt} \qquad on\ r = a\ (M\ is\ a\ constant)$$

$$= 0 \qquad on\ r = b$$

$$\sigma_r = 0 \qquad on\ r = a\ and\ r = b$$

where a and b are the outer and inner radii of the annular disk. The non-homogeneity of the material of the disk is assumed to obey the laws

$$E = E_o e^{\beta x}$$

$$\rho = \rho_o e^{\beta x}$$

where $x = r/b$ and E_0 and ρ_0 are constants and β is a real number.

Substitutions from (2) and (5) into (1), leads to two coupled differential equations in U_r and U_θ. The nature of angular velocity Ω in (3) suggests U_r and U_θ in the following forms

$$U_r = F_1(x)\, e^{-2p\tau/b} \tag{6}$$

$$U_\theta = F_2(x)\, e^{-p\tau/b}$$

With such a choice, the field equations in non-dimensional variable x may be written as

$$\frac{d^2F_1}{dx^2} + \left(\beta + \frac{1}{x}\right)\frac{dF_1}{dx} + \left(\frac{\upsilon\beta}{x} - \frac{1}{x^2} - K_2\right)F_1 = -K_1 x \tag{7}$$

and

$$\frac{d^2F_2}{dx^2} + \left(\beta + \frac{1}{x}\right)\frac{dF_2}{dx} - \left(\frac{\beta}{x} + \frac{1}{x^2} + K_4\right)F = -K_3 x \tag{8}$$

where

$$K_1 = \rho_o\, \Omega_o^2 b^2 \left(1-\upsilon^2\right)/E_o$$

$$K_2 = 4\, \rho_o\, p^2\, b^2 \left(1-\upsilon^2\right)/E_o\, t_o^2$$

$$K_3 = 2\, \rho_o\, \Omega_o\, b^2 p\, (1+\upsilon)/E_o\, t_o$$

$$K_4 = 2\, \rho_o\, b^2 p^2\, (1+\upsilon)/E_o\, t_o^2$$

Thus our problem is to solve (7) and (8) under boundary conditions (4).

Solution of the problem

Setting $F_1 = x. V_1$

and $F_2 = x. V_2$, equations (7) and (8) become

$$x\frac{d^2 V_1}{dx^2} + (3+\beta x)\frac{dV_1}{dx} + \left[\beta\,(1=\upsilon) - K_2 x\right] V_1 = -K_1 x \tag{9}$$

$$x\frac{d^2 V_2}{dx^2} + (3+\beta x)\frac{dV_2}{dx} - (K_4 x)\, V_2 = -K_3 x \tag{10}$$

The complementary equations of (9) and (10) may be easily transformed into confluent hypergeometric differential equations by suitable substitutions. Since the solution of such equation is known the general solution of (9) (10) may be given by

$$F_1 = x.\, e^{s_1 x}\left[A_1 \phi_1\,(C_1 x) + B_1\, \psi_1 (C_1 x)\right] + x\eta_1\,(x) \tag{11}$$

and

$$F_2 = x.\, e^{s_2 x}\left[A_2 \phi_2\,(C_2 x) + B_2\, \psi_2 (C_2 x)\right] + x\eta_2\,(x) \tag{12}$$

Where ϕ_1, ψ_1 and ϕ_2, ψ_2 are respectively known solutions of the confluent hypergeometric equations, [4]

$$\xi\frac{d^2 y}{d\xi^2} + \left(3-\xi\right)\frac{dy}{d\xi} - \alpha_1 y = 0$$

and

$$\xi \frac{d^2y}{d\xi^2}+\left(3-\xi\right)\frac{dy}{d\xi}-\alpha_2 y=0$$

Where

$\alpha_1 = (3s_1 + \beta(1 + \upsilon)]/[2s_1 + \beta], C_1 = -(2s_1 + \beta)$

$\alpha_2 = (3s_2 / [2s_2 + \beta], C_2 = -(2s_2 + \beta)$

s_1 *is a root of* $\theta^2 + \beta\theta - K_2 = 0$, *while* s_2 *is that of* $\theta^2 + \beta\theta - K_4 = 0$

The functions η_1 (x) and η_2 (x) appearing in (11) and (12) are given by

$$\eta_1(x) = \sum_{n=2}^{\infty} a_n x^n$$

$$\eta_2(x) = +K_3/K_4$$

where

$a_2 = -K_1/8$

$a_3 = \beta(3 + \upsilon).K_1/120$

and other a_i's can be calculated from the relation

$(n+1)(n+3)a_{n+1} + \beta(1 + \upsilon + n)a_n - K_2 a_{n-1} = 0$ *for* $n \geq 3.$

Knowing F_1 and F_2, we can find U_r and U_θ and U_θ from (6). Using these U_r and U_θ, we get from (2)

$$\sigma_r = \frac{E_0 e^{-2p\tau}}{b\left(1-\upsilon^2\right)}\left[A_1R_1(x)+B_1R_2(x)+R_3(x)\right]$$

$$\sigma_r = \frac{E_0 e^{-2p r}}{b\left(1-\upsilon^2\right)}\left[A_1R_4(x)+B_1B_5(x)+R_6(x)\right]$$

and

$$\tau_{r\theta} = M \, . \, e^{-p\,\tau} \left[A_2 R_7 \langle x \rangle + B_2 R_8 (x) \right]$$

.where

$$R_1 (x) = [\, (1 + \upsilon + s_1 x)\, \phi_1 (C_1 x) + C_1 x\, \phi'_1 (C_1 x)\,]\, e^{(\beta + s_1)\, x}$$

$$R_2 (x) = [\, (1 + \upsilon + s_1 x)\, \psi_1 (C_1 x) + C_1 x\, \psi'_1 (C_1 x)\,]\, e^{(\beta + s_1)\, x}$$

$$R_3 (x) = [\, (1 + \upsilon) + \eta_1 (x) + x .\, \eta'_1 (x)\ ]\ e^{\beta x}$$

$$R_4 (x) = [\, (1 + \upsilon + \ \upsilon x s_1)\, \phi_1 (C_1 x) + \upsilon x C_1 . \phi'_1 (C_1 x)\,]\, e^{(\beta + s_1)\, x}$$

$$R_5 (x) = [\, (1 + \upsilon + \ \upsilon x s_1)\, \psi'_1 (C_1 x) + \upsilon x C_1 .\ \psi'_1 (C_1 x)\,]\, e^{(\beta + s_1)}$$

x

$$R_6 (x) = [\, (1 + \upsilon)\, \eta_1 (x) + x \upsilon\ \eta'_1 (x)\,]\, e^{(\beta x)}$$

$$R_7 (x) = [\, (x\ s_2)\, \phi_2 (C_2 x) + .\, C_2\ \phi'_1 (C_2 x)\,]\, e^{(\beta + s_2)\, x}$$

$$R_8 (x) = [\, (x\ s_2)\, \psi_2 (C_2 x) + .\, C_2\ \psi'_1 (C_2 x)\,]\, e^{(\beta + s_2)\, x}$$

The constants A_1, B_1, A_2, B_2 *in (13), (14) and (15) may be* determined from the boundary conditions (4), as

$$A_1 = [R_2 (1) R_3 (x_o) - R_2 (x_o) R_3 (1)\,] / \Delta_1$$

$$B_1 = [R_1 (x_o) R_3 (1) - R_1 (1) R_3 (x_o)\,] / \Delta_1$$

$$A_2 = [+ R_8 (1)\,] / \Delta_2$$

$$B_2 = - R_7 (1)\,] / \Delta_2$$

where $x_o = a / b$

$$\Delta_1 = R_1 (1) R_2 (x_o) - R_1 (x_o) R_2 (1)$$

$$\Delta_2 = R_7 (1) R_8 (x_o) - R_7 (x_o) R_8 (1)$$

Thus knowing A_1, B_1, A_2, B_2, we get the complete expressions for the components of stresses from (13),(14) and (15).

Numerical results :

To study the nature of stress distribution and also the effect of nonhomogeneity on the stresses we compute radial and tangential stress components along with shearing stress component at different positions in the disk. In our computation we have assumed $p = 1/2, \Omega_o t_o = 1, \upsilon = 1/3$ *and* $\rho_o \Omega^2{}_o b^2 = E_o$

We plot variations of

$$P = \left[b\left(1-\upsilon^2\right) e^2 p\tau \Big/ E_0 \right] \sigma_r$$

$$Q = -\left[b\left(1-\upsilon^2\right) e^2 p\tau \Big/ E_0 \right] \sigma_\theta$$

$$R = \left[e^{p\tau} \Big/ M \right] \tau_{r\theta}$$

with position in Figure 1, 2 and 3.

The results in the associated homogeneous case ($\beta = 0$) have been shown in broken lines. The effect of non-homogeneity is clear from the graph.

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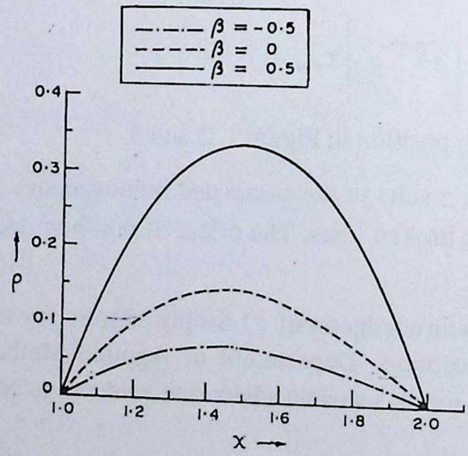

Fig. 1 Variation of radial stress with position (for fixed τ).

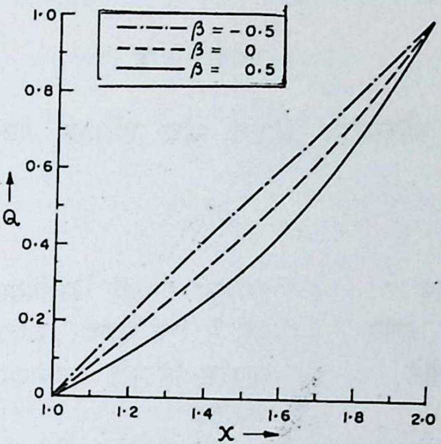

Fig. 2. Variation of shearing stress with position (for fixed τ).

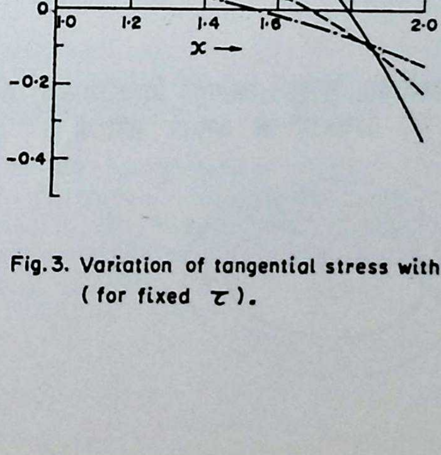

Fig. 3. Variation of tangential stress with position (for fixed τ).

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फार्म – ४

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